Scalars and vectors

The quantities measured in Physics may be divided into two groups:

(a) scalars - these are quantities that have magnitude (size) only.

Examples of scalars are length, speed, mass, density, energy, power, temperature, charge, potential difference

(b) vectors - these are quantities that have direction as well as magnitude.
Examples of vectors are displacement, force, torque, velocity, acceleration, momentum, electric current, magnetic flux density, electric field

Scalars may be added together by simple arithmetic but when two or more vectors are added together their direction must be taken into account as well.

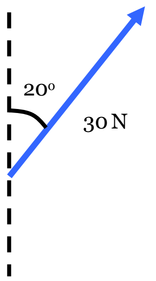


Figure 1

A vector may be represented by a line, the length of the line being the magnitude of the vector and the direction of the line the direction of the vector.

For example Figure 1 shows a force of 30 N acting at 20^o to the vertical of the page.

A simple comparison between a vector and a scalar is shown by Figure 2. This shows the distance moved by a soccer referee during the match. The vector represents the displacement between the starting position (A) and his position at the end of the game (B) while the wavy line (a scalar) is the distance that the referee has actually run during the game (much further!).

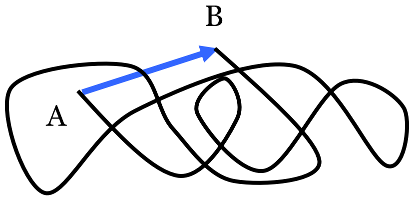
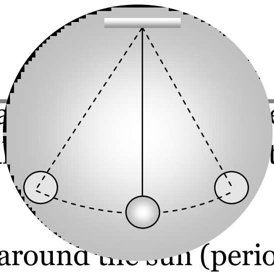


Figure 2

When it is said that a vector has a direction it means that either it is moving in a certain direction (as in a velocity) or that it could produce movement in a certain direction (as in a force or a magnetic field).

Simple Harmonic Motion



Periodic Motion

A motion, which repea[illegible]er again after a regular interval of time is called a periodic motion and the fixed interval [illegible] the motion is repeated is called period of the motion.

Examples :

(i) Revolution of earth around [illegible] sun (period one year)

(ii) Rotation of earth about its polar axis (period one day)

(iii) Motion of hour's hand of a clock (period 12-hour)

(iv) Motion of minute's hand of a clock (period 1-hour)

(v) Motion of second's hand of a clock (period 1-minute)

(vi) Motion of moon around the earth (period 27.3 days)

15.2 Oscillatory or Vibratory Motion

Oscillatory or vibratory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time. In such a motion, the body is confined with in well-defined limits on either side of mean position.

Oscillatory motion is also called as harmonic motion.

Example :

(i) The motion of the pendulum of a wall clock.

(ii) The motion of a load attached to a spring, when it is pulled and then released.

(iii) The motion of liquid contained in U- tube when it is compressed once in one limb and left to itself.

(iv) A loaded piece of wood floating over the surface of a liquid when pressed down and then released executes oscillatory motion.

15.3 Harmonic and Non-harmonic Oscillation

Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (*i.e.* sine or cosine function). *Example* : $y = a\sin\omega t$ or $y = a\cos\omega t$

Non-harmonic oscillation is that oscillation which can not be expressed in terms of single harmonic function. It is a combination of two or more than two harmonic oscillations. *Example* : $y = a\sin\omega t + b\sin 2\omega t$

15.4 Some Important Definitions

(1) **Time period :** It is the least interval of time after which the periodic motion of a body repeats itself.

S.I. units of time period is second.

(2) **Frequency :** It is defined as the number of periodic motions executed by body per second. S.I unit of frequency is hertz (Hz).

(3) **Angular Frequency :** Angular frequency of a body executing periodic motion is equal to product of frequency of the body with factor 2π. Angular frequency $\omega = 2\pi n$

S.I. units of ω is Hz [S.I.] ω also represents angular velocity. In that case unit will be *rad/sec*.

(4) **Displacement :** In general, the name displacement is given to a physical quantity which undergoes a change with time in a periodic motion.

Examples :

(i) In an oscillation of a loaded spring, displacement variable is its deviation from the mean position.

(ii) During the propagation of sound wave in air, the displacement variable is the local change in pressure

(iii) During the propagation of electromagnetic waves, the displacement variables are electric and magnetic fields, which vary periodically.

(5) **Phase :** phase of a vibrating particle at any instant is a physical quantity, which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

In oscillatory motion the phase of a vibrating particle is the argument of *sine* or *cosine* function involved to represent the generalised equation of motion of the vibrating particle.

$$y = a\sin\theta = a\sin(\omega t + \phi_0) \quad \text{here, } \theta = \omega t + \phi_0 = \text{phase of vibrating particle.}$$

(i) Initial phase or epoch : It is the phase of a vibrating particle at $t = 0$.

In $\theta = \omega t + \phi_0$, when $t = 0$; $\theta = \phi_0$ here, ϕ_0 is the angle of epoch.

(ii) Same phase : Two vibrating particle are said to be in same phase, if the phase difference between them is an even multiple of π or path difference is an even multiple of $(\lambda / 2)$ or time interval is an even multiple of $(T / 2)$ because 1 time period is equivalent to $2\pi\, rad$ or 1 wave length (λ)

(iii) Opposite phase : When the two vibrating particles cross their respective mean positions at the same time moving in opposite directions, then the phase difference between the two vibrating particles is 180^o

Opposite phase means the phase difference between the particle is an odd multiple of π (say π, 3π, 5π, 7π.....) or the path difference is an odd multiple of λ (say $\frac{\lambda}{2}, \frac{3\lambda}{2}$,.......) or the time interval is an odd multiple of $(T / 2)$.

(iv) Phase difference : If two particles performs S.H.M and their equation are

$$y_1 = a\sin(\omega t + \phi_1) \text{ and } y_2 = a\sin(\omega t + \phi_2)$$

then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

15.5 Simple Harmonic Motion

Simple harmonic motion is a special type of periodic motion, in which a particle moves to and fro repeatedly about a mean position under a restoring force which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean position at that instant.

Restoring force $\propto$ Displacement of the particle from mean position.

$$F \propto -x$$

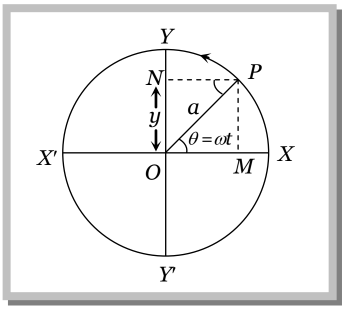
$$F = -kx$$

Where k is known as force constant. Its S.I. unit is Newton/meter and dimension is $[MT^{-2}]$.

15.6 Displacement in S.H.M.

The displacement of a particle executing S.H.M. at an instant is defined as the distance of particle from the mean position at that instant.

As we know that simple harmonic motion is defined as the projection of uniform circular motion on any diameter of circle of reference. If the projection is taken on y-axis.



then from the figure $y = a\sin\omega t$

$$y = a\sin\frac{2\pi}{T}t$$

$$y = a\sin 2\pi n t$$

$$y = a\sin(\omega t \pm \phi)$$

where a = Amplitude, ω = Angular frequency, t = Instantaneous time,

T = Time period, n = Frequency and ϕ = Initial phase of particle

If the projection of P is taken on X-axis then equations of S.H.M. can be given as

$$x = a\cos(\omega t \pm \phi)$$

$$x = a\cos\left(\frac{2\pi}{T}t \pm \phi\right)$$

$$x = a\cos(2\pi n t \pm \phi)$$

Important points

(i) $y = a\sin\omega t$ when the time is noted from the instant when the vibrating particle is at mean position.

(ii) $y = a\cos\omega t$ when the time is noted from the instant when the vibrating particle is at extreme position.

(iii) $y = a\sin(\omega t \pm \phi)$ when the vibrating particle is ϕ phase leading or lagging from the mean position.

(iv) Direction of displacement is always away from the equilibrium position, particle either is moving away from or is coming towards the equilibrium position.

(v) If t is given or phase (θ) is given, we can calculate the displacement of the particle.

If $t = \frac{T}{4}$ (or $\theta = \frac{\pi}{2}$) then from equation $y = a\sin\frac{2\pi}{T}t$, we get $y = a\sin\frac{2\pi}{T}\frac{T}{4} = a\sin\left(\frac{\pi}{2}\right) = a$

Similarly if $t = \frac{T}{2}$ (or $\theta = \pi$) then we get $y = 0$

Sample problems based on Displacement

Problem 1. A simple harmonic oscillator has an amplitude A and time period T. The time required by it to travel from $x = A$ to $x = A/2$ is **[CBSE 1992; SCRA 1996]**

(a) $T/6$ (b) $T/4$ (c) $T/3$ (d) $T/2$

Solution : (a) Because the S.H.M. starts from extreme position so $y = a\cos\omega t$ form of S.H.M. should be used.

$$\frac{A}{2} = A\cos\frac{2\pi}{T}t \Rightarrow \cos\frac{\pi}{3} = \cos\frac{2\pi}{T}t \Rightarrow t = T/6$$

Problem 2. A mass m = 100 *gms* is attached at the end of a light spring which oscillates on a friction less horizontal table with an amplitude equal to 0.16 meter and the time period equal to 2 *sec*. Initially the mass is released from rest at t = 0 and displacement x = – 0.16 *meter*. The expression for the displacement of the mass at any time (t) is **[MP PMT 1995]**

(a) $x = 0.16\cos(\pi t)$ (b) $x = -0.16\cos(\pi t)$ (c) $x = 0.16\cos(\pi t + \pi)$ (d) $x = -0.16\cos(\pi t + \pi)$

Solution : (b) Standard equation for given condition

$$x = a\cos\frac{2\pi}{T}t \Rightarrow x = -0.16\cos(\pi t) \qquad [\text{As } a = -0.16\ meter,\ T = 2\ sec]$$

Problem 3. The motion of a particle executing S.H.M. is given by $x = 0.01\sin 100\pi(t + .05)$. Where x is in meter and time t is in seconds. The time period is

(a) 0.01 *sec* (b) 0.02 *sec* (c) 0.1 *sec* (d) 0.2 *sec*

Solution : (b) By comparing the given equation with standard equation $y = a\sin(\omega t + \phi)$

$$\omega = 100\pi \quad \text{so } T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = 0.02\ sec$$

Problem 4. Two equations of two S.H.M. are $x = a\sin(\omega t - \alpha)$ and $y = b\cos(\omega t - \alpha)$. The phase difference between the two is **[MP PMT 1985]**

(a) 0^o (b) α^o (c) 90^o (d) 180^o

Solution : (c) $x = a\sin(\omega t - \alpha)$ and $y = b\cos(\omega t - \alpha) = b\sin(\omega t - \alpha + \pi/2)$

Now the phase difference = $(\omega t - \alpha + \frac{\pi}{2}) - (\omega t - \alpha) = \pi/2 = 90^o$

15.7 Velocity in S.H.M.

Velocity of the particle executing S.H.M. at any instant, is defined as the time rate of change of its displacement at that instant.

In case of S.H.M. when motion is considered from the equilibrium position

$$y = a\sin\omega t$$

so $$v = \frac{dy}{dt} = a\omega\cos\omega t$$

$\therefore$ $$v = a\omega\cos\omega t \qquad \text{......(i)}$$

or $$v = a\omega\sqrt{1 - \sin^2\omega t} \qquad [\text{As } \sin\omega t = y/a]$$

or $$v = \omega\sqrt{a^2 - y^2} \qquad \text{.....(ii)}$$

Important points

(i) In S.H.M. velocity is maximum at equilibrium position.

From equation (i) $v_{max} = a\omega$ when $|\cos\omega t| = 1$ *i.e.* $\theta = \omega t = 0$

from equation (ii) $v_{max} = a\omega$ when $y = 0$

(ii) In S.H.M. velocity is minimum at extreme position.

From equation (i) $v_{min} = 0$ when $|\cos\omega t| = 0$ *i.e* $\theta = \omega t = \frac{\pi}{2}$

From equation (ii) $v_{min} = 0$ when $y = a$

(iii) Direction of velocity is either towards or away from mean position depending on the position of particle.

Sample problems based on Velocity

Problem 5. A body is executing simple harmonic motion with an angular frequency 2 *rad/sec*. The velocity of the body at 20 *mm* displacement. When the amplitude of motion is 60 *mm* is **[AFMC 1998]**

(a) 40 *mm/sec* (b) 60 *mm/sec* (c) 113 *mm/sec* (d) 120 *mm/sec*

Solution : (c) $v = \omega\sqrt{a^2 - y^2} = 2\sqrt{(60)^2 - (20)^2} = 113$ *mm/sec*

Problem 6. A body executing S.H.M. has equation $y = 0.30\sin(220\,t + 0.64)$ in *meter*. Then the frequency and maximum velocity of the body is

(a) $35 Hz, 66 m/s$ (b) $45 Hz, 66 m/s$ (c) $58 Hz, 113 m/s$ (d) $35 Hz, 132 m/s$

Solution : (a) By comparing with standard equation $y = a\sin(\omega t + \phi)$ we get $a = 0.30;\ \omega = 220$

$\therefore\ 2\pi n = 220 \Rightarrow n = 35 Hz$ so $v_{max} = a\omega = 0.3 \times 220 = 66 m/s$

Problem 7. A particle starts S.H.M. from the mean position. Its amplitude is *A* and time period is *T*. At the time when its speed is half of the maximum speed. Its displacement *y* is

(a) $A/2$ (b) $A/\sqrt{2}$ (c) $A\sqrt{3}/2$ (d) $2A/\sqrt{3}$

Solution : (c) $v = \omega\sqrt{a^2 - y^2} \Rightarrow \frac{a\omega}{2} = \omega\sqrt{a^2 - y^2} \Rightarrow \frac{a^2}{4} = a^2 - y^2 \Rightarrow y = \frac{\sqrt{3}A}{2}$ [As $v = \frac{v_{\max}}{2} = \frac{a\omega}{2}$]

Problem 8. A particle perform simple harmonic motion. The equation of its motion is $x = 5\sin(4t - \frac{\pi}{6})$. Where *x* is its displacement. If the displacement of the particle is 3 units then its velocity is **[MP PMT 1994]**

(a) $2\pi/3$ (b) $5\pi/6$ (c) 20 (d) 16

Solution : (d) $v = \omega\sqrt{a^2 - y^2} = 4\sqrt{5^2 - 3^2} = 16$ [As $\omega = 4, a = 5, y = 3$]

Problem 9. A simple pendulum performs simple harmonic motion about *x* = 0 with an amplitude (*A*) and time period (*T*). The speed of the pendulum at $x = \frac{A}{2}$ will be **[MP PMT 1987]**

(a) $\frac{\pi A\sqrt{3}}{T}$ (b) $\frac{\pi A}{T}$ (c) $\frac{\pi A\sqrt{3}}{2T}$ (d) $\frac{3\pi^2 A}{T}$

Solution : (a) $v = \omega\sqrt{a^2 - y^2} \Rightarrow v = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{\pi A\sqrt{3}}{T}$ [As *y* = A/2]

Problem 10. A particle is executing S.H.M. if its amplitude is 2 *m* and periodic time 2 *seconds*. Then the maximum velocity of the particle will be

(a) 6π (b) 4π (c) 2π (d) π

Solution : (c) $v_{max} = a\omega = a\frac{2\pi}{T} = 2\frac{2\pi}{2} \Rightarrow v_{max} = 2\pi$

Problem 11. A S.H.M. has amplitude '*a*' and time period *T*. The maximum velocity will be **[MP PMT 1985]**

(a) $\frac{4a}{T}$ (b) $\frac{2a}{T}$ (c) $2\pi\sqrt{\frac{a}{T}}$ (d) $\frac{2\pi a}{T}$

Solution : (d) $v_{\max} = a\omega = \frac{a2\pi}{T}$

Problem 12. A particle executes S.H.M. with a period of 6 second and amplitude of 3 *cm* its maximum speed in *cm/sec* is

[AIIMS 1982]

(a) $\pi / 2$ (b) π (c) 2π (d) 3π

Solution : (b) $v_{max} = a\omega = a\frac{2\pi}{T} = 3\frac{2\pi}{6} \Rightarrow v_{max} = \pi$

***Problem* 13.** A body of mass 5 *gm* is executing S.H.M. about a point with amplitude 10 *cm*. Its maximum velocity is 100 *cm/sec*. Its velocity will be 50 *cm/sec*, at a distance **[CPMT 1976]**

(a) 5 (b) $5\sqrt{2}$ (c) $5\sqrt{3}$ (d) $10\sqrt{2}$

Solution : (c) $v_{max} = a\omega = 100\ cm/\sec$ and $a = 10\ cm$ so $\omega = 10\ rad/\sec$.

$\therefore\ v = \omega\sqrt{a^2 - y^2} \Rightarrow 50 = 10\sqrt{10^2 - y^2} \Rightarrow y = 5\sqrt{3}$

15.8 Acceleration in S.H.M.

The acceleration of the particle executing S.H.M. at any instant, is defined as the rate of change of its velocity at that instant. So acceleration $A = \frac{dv}{dt} = \frac{d}{dt}(a\omega\cos\omega t)$

$A = -\omega^2 a\sin\omega t$ ......(i)

$A = -\omega^2 y$ ......(ii) [As $y = a\sin\omega t$]

Important points

(i) In S.H.M. as $|\text{Acceleration}| = \omega^2 y$ is not constant. So equations of translatory motion can not be applied.

(ii) In S.H.M. acceleration is maximum at extreme position.

From equation (i) $|A_{max}| = \omega^2 a$ when $|\sin\omega t| = \text{maximum} = 1$ *i.e.* at $t = \frac{T}{4}$ or $\omega t = \frac{\pi}{2}$

From equation (ii) $|A_{max}| = \omega^2 a$ when $y = a$

(iii) In S.H.M. acceleration is minimum at mean position

From equation (i) $A_{min} = 0$ when $\sin\omega t = 0$ *i.e.* at $t = 0$ or $t = \frac{T}{2}$ or $\omega t = \pi$

From equation (ii) $A_{min} = 0$ when $y = 0$

(iv) Acceleration is always directed towards the mean position and so is always opposite to displacement

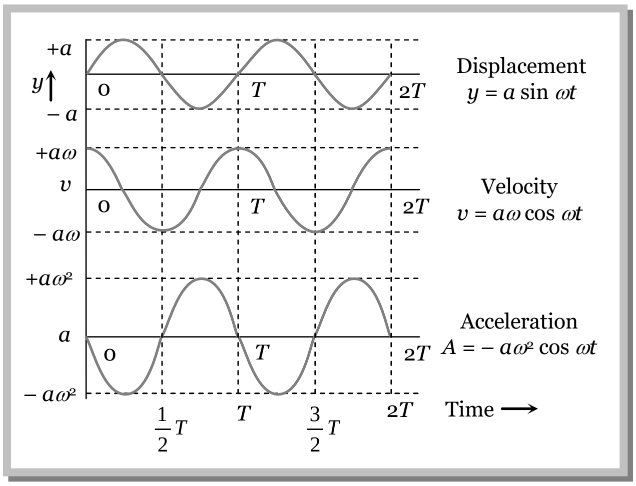
i.e., $A \propto -y$

15.9 Comparative Study of Displacement, Velocity and Acceleration

Displacement $y = a\sin\omega t$

Velocity $v = a\omega\cos\omega t = a\omega\sin(\omega t + \frac{\pi}{2})$

Acceleration $A = -a\omega^2\sin\omega t = a\omega^2\sin(\omega t + \pi)$



From the above equations and graphs we can conclude that.

(i) All the three quantities displacement, velocity and acceleration show harmonic variation with time having same period.

(ii) The velocity amplitude is ω times the displacement amplitude

(iii) The acceleration amplitude is ω^2 times the displacement amplitude

(iv) In S.H.M. the velocity is ahead of displacement by a phase angle $\pi/2$

(v) In S.H.M. the acceleration is ahead of velocity by a phase angle $\pi/2$

(vi) The acceleration is ahead of displacement by a phase angle of π

(vii) Various physical quantities in S.H.M. at different position :

Physical quantities	Equilibrium position (y = 0)	Extreme Position ($y = \pm a$)
Displacement $y = a\sin\omega t$	Minimum (Zero)	Maximum (a)
Velocity $v = \omega\sqrt{a^2 - y^2}$	Maximum ($a\omega$)	Minimum (Zero)
Acceleration $\lvert A \rvert = \omega^2 y$	Minimum (Zero)	Maximum ($\omega^2 a$)

15.10 Energy in S.H.M.

A particle executing S.H.M. possesses two types of energy : Potential energy and Kinetic energy

(1) **Potential energy :** This is an account of the displacement of the particle from its mean position.

The restoring force $F = -ky$ against which work has to be done

So $$U = -\int dw = -\int_0^x F dx = \int_0^y ky\, dy = \frac{1}{2}ky^2$$

$\therefore$ potential Energy $$U = \frac{1}{2}m\omega^2 y^2 \qquad [\text{As } \omega^2 = k/m]$$

$$U = \frac{1}{2}m\omega^2 a^2 \sin^2 \omega t \qquad [\text{As } y = a\sin\omega t]$$

Important points

(i) Potential energy maximum and equal to total energy at extreme positions

$$U_{max} = \frac{1}{2}ka^2 = \frac{1}{2}m\omega^2 a^2 \qquad \text{when } y = \pm a;\ \omega t = \pi/2;\ t = T/4$$

(ii) Potential energy is minimum at mean position

$$U_{min} = 0 \qquad \text{when } y = 0;\ \omega t = 0;\ t = 0$$

(2) **Kinetic energy :** This is because of the velocity of the particle

Kinetic Energy $$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}ma^2\omega^2 \cos^2 \omega t \qquad [\text{As } v = a\omega\cos\omega t]$$

$$K = \frac{1}{2}m\omega^2(a^2 - y^2) \qquad [\text{As } v = \omega\sqrt{a^2 - y^2}]$$

(i) Kinetic energy is maximum at mean position and equal to total energy at mean position.

$$K_{max} = \frac{1}{2}m\omega^2 a^2 \qquad \text{when } y=0;\ t=0;\ \omega t = 0$$

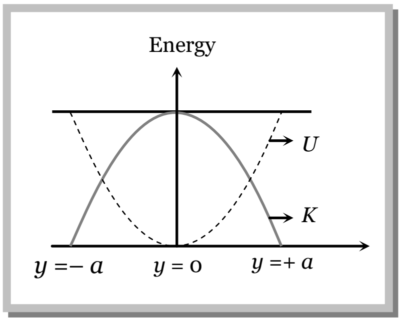
(ii) Kinetic energy is minimum at extreme position.

$$K_{min} = 0 \qquad \text{when } y=a;\ t=T/4,\ \omega t=\pi/2$$

(3) **Total energy :** Total mechanical energy = Kinetic energy + Potential energy

$$E=\frac{1}{2}m\omega^2(a^2-y^2)+\frac{1}{2}m\omega^2y^2=\frac{1}{2}m\omega^2a^2$$

Total energy is not a position function *i.e.* it always remains constant.



(4) **Energy position graph :** Kinetic energy $(K) = \frac{1}{2}m\omega^2(a^2-y^2)$

Potential Energy $(U) = \frac{1}{2}m\omega^2y^2$

Total Energy $(E) = \frac{1}{2}m\omega^2a^2$

It is clear from the graph that

(i) Kinetic energy is maximum at mean position and minimum at extreme position

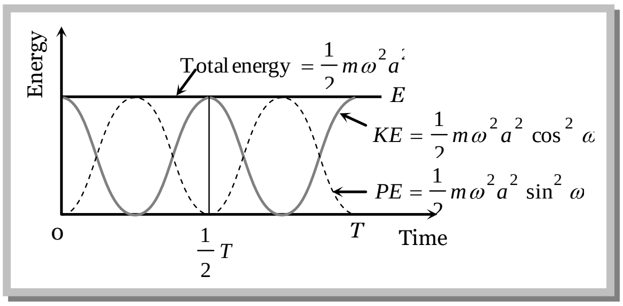
(ii) Potential energy is maximum at extreme position and minimum at mean position

(iii) Total energy always remains constant.

(5) Kinetic Energy $\quad K=\frac{1}{2}m\omega^2a^2\cos^2\omega t=\frac{1}{4}m\omega^2a^2(1+\cos 2\omega t)=\frac{1}{2}E(1+\cos\omega' t)$

Potential Energy $\quad U=\frac{1}{2}m\omega^2a^2\sin^2\omega t=\frac{1}{4}m\omega^2a^2(1-\cos 2\omega t)=\frac{1}{2}E(1-\cos\omega' t)$

where $\omega'=2\omega$ and $E=\frac{1}{2}m\omega^2a^2$



i.e. in S.H.M., kinetic energy and potential energy vary periodically with double the frequency of S.H.M. (*i.e.* with time period $T'=T/2$)

From the graph we note that potential energy or kinetic energy completes two vibrations in a time during which S.H.M. completes one vibration. Thus the frequency of potential energy or kinetic energy double than that of S.H.M.

Sample problems based on Energy

***Problem* 14.** A particle is executing simple harmonic motion with frequency *f*. The frequency at which its kinetic energy changes into potential energy is

(a) $f/2$ (b) f (c) $2f$ (d) $4f$

Solution : (c)

***Problem* 15.** When the potential energy of a particle executing simple harmonic motion is one-fourth of the maximum value during the oscillation, its displacement from the equilibrium position in terms of amplitude '*a*' is

[CBSE 1993; MP PMT 1994; MP PET 1995, 96; MP PMT 2000]

(a) $a/4$ (b) $a/3$ (c) $a/2$ (d) $2a/3$

Solution : (c) According to problem potential energy $= \frac{1}{4}$ maximum Energy

$\Rightarrow \frac{1}{2} m\omega^2 y^2 = \frac{1}{4}\left(\frac{1}{2} m\omega^2 a^2\right) \Rightarrow y^2 = \frac{a^2}{4} \Rightarrow y = a/2$

Problem 16. A particle of mass 10 *grams* is executing S.H.M. with an amplitude of 0.5 *meter* and circular frequency of 10 *radian/sec*. The maximum value of the force acting on the particle during the course of oscillation is

[MP PMT 2000]

(a) 25 *N* (b) 5 *N* (c) 2.5 *N* (d) 0.5 *N*

Solution : (d) Maximum force = mass × maximum acceleration $= m\omega^2 a = 10 \times 10^{-3}(10)^2(0.5) = 0.5\,N$

Problem 17. A body is moving in a room with a velocity of 20 *m/s* perpendicular to the two walls separated by 5 meters. There is no friction and the collision with the walls are elastic. The motion of the body is **[MP PMT 19**

(a) Not periodic (b) Periodic but not simple harmonic

(c) Periodic and simple harmonic (d) Periodic with variable time period

Solution : (b) Since there is no friction and collision is elastic therefore no loss of energy take place and the body strike again and again with two perpendicular walls. So the motion of the ball is periodic. But here, there is no restoring force. So the characteristics of S.H.M. will not satisfied.

Problem 18. Two particles executes S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions. Each time their displacement is half of their amplitude. The phase difference between them is

(a) 30° (b) 60° (c) 90° (d) 120°

Solution : (d) Let two simple harmonic motions are $y = a\sin\omega t$ and $y = a\sin(\omega t + \phi)$

In the first case $\frac{a}{2} = a\sin\omega t \Rightarrow \sin\omega t = 1/2$ $\therefore$ $\cos\omega t = \frac{\sqrt{3}}{2}$

In the second case $\frac{a}{2} = a\sin(\omega t + \phi)$

$\Rightarrow \frac{1}{2} = [\sin\omega t.\cos\phi + \cos\omega t\sin\phi] \Rightarrow \frac{1}{2} = \left[\frac{1}{2}\cos\phi + \frac{\sqrt{3}}{2}\sin\phi\right]$

$\Rightarrow 1 - \cos\phi = \sqrt{3}\sin\phi \Rightarrow (1-\cos\phi)^2 = 3\sin^2\phi \Rightarrow (1-\cos\phi)^2 = 3(1-\cos^2\phi)$

By solving we get $\cos\phi = +1$ or $\cos\phi = -1/2$

i.e. $\phi = 0$ or $\phi = 120^o$

Problem 19. The acceleration of a particle performing S.H.M. is 12 *cm/sec*² at a distance of 3 *cm* from the mean position. Its time period is

(a) 0.5 *sec* (b) 1.0 *sec* (c) 2.0 *sec* (d) 3.14 *sec*

Solution : (d) $A = \omega^2 y \Rightarrow \omega = \sqrt{\frac{A}{y}} = \sqrt{\frac{12}{3}} = 2$; but $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi = 3.14$

Problem 20. A particle of mass 10 *gm* is describing S.H.M. along a straight line with period of 2 *sec* and amplitude of 10 *cm*. Its kinetic energy when it is at 5 cm. From its equilibrium position is

(a) $37.5\pi^2 erg$ (b) $3.75\pi^2 erg$ (c) $375\pi^2 erg$ (d) $0.375\pi^2 erg$

Solution : (c) Kinetic energy $= \frac{1}{2} m\omega^2(a^2 - y^2) = \frac{1}{2} 10\,\frac{4\pi^2}{4}(10^2 - 5^2) = 375\pi^2 ergs$.

Problem 21. The total energy of the body executing S.H.M. is E. Then the kinetic energy when the displacement is half of the amplitude is **[RPET 1996]**

(a) $E/2$ (b) $E/4$ (c) $3E/4$ (d) $\sqrt{3}E/4$

Solution : (c) Kinetic energy $=\frac{1}{2}m\omega^2(a^2-y^2)=\frac{1}{2}m\omega^2\left(a^2-\frac{a^2}{4}\right)=\frac{3}{4}\left(\frac{1}{2}m\omega^2a^2\right)=\frac{3E}{4}$ [As $y=\frac{a}{2}$]

Problem 22. A body executing simple harmonic motion has a maximum acceleration equal to 24 m/sec^2 and maximum velocity equal to 16 $meter/sec$. The amplitude of simple harmonic motion is **[MP PMT 1995]**

(a) $\frac{32}{3}\,meters$ (b) $\frac{3}{32}\,meters$ (c) $\frac{1024}{9}\,meters$ (d) $\frac{64}{9}\,meters$

Solution : (a) Maximum acceleration $\omega^2a=24$ .....(i)

and maximum velocity $a\omega=16$ ....(ii)

Dividing (i) by (ii) $\omega=\frac{3}{2}$

Substituting this value in equation (ii) we get $a=32/3\,meter$

Problem 23. The displacement of an oscillating particle varies with time (in seconds) according to the equation. $y(cm)=\sin\frac{\pi}{2}\left(\frac{t}{2}+\frac{1}{3}\right)$. The maximum acceleration of the particle approximately

(a) 5.21 cm/sec^2 (b) 3.62 cm/sec^2 (c) 1.81 cm/sec^2 (d) 0.62 cm/sec^2

Solution : (d) By comparing the given equation with standard equation, $y=a\sin(\omega t+\phi)$

We find that $a=1$ and $\omega=\pi/4$

Now maximum acceleration $=\omega^2a=\left(\frac{\pi^2}{4}\right)=\left(\frac{3.14}{4}\right)^2=0.62\,cm/\sec^2$

Problem 24. The potential energy of a particle executing S.H.M. at a distance x from the mean position is proportional to

[Roorkee 1992]

(a) $\sqrt{x}$ (b) x (c) x^2 (d) x^3

Solution : (c)

Problem 25. The kinetic energy and potential energy of a particle executing S.H.M. will be equal, when displacement is (amplitude = a) **[MP PMT 1987; CPMT 1990]**

(a) $a/2$ (b) $a\sqrt{2}$ (c) $a/\sqrt{2}$ (d) $\frac{a\sqrt{2}}{3}$

Solution : (c) According to problem Kinetic energy = Potential energy $\Rightarrow\frac{1}{2}m\omega^2(a^2-y^2)=\frac{1}{2}m\omega^2y^2$

$\Rightarrow a^2-y^2=y^2\ \therefore\ y=a/\sqrt{2}$

Problem 26. The phase of a particle executing S.H.M. is $\frac{\pi}{2}$ when it has

(a) Maximum velocity (b) Maximum acceleration (c) Maximum energy (d)

Solution : (b, d) Phase $\pi/2$ means extreme position. At extreme position acceleration and displacement will be maximum.

Problem 27. The displacement of a particle moving in S.H.M. at any instant is given by $y=a\sin\omega t$. The acceleration after time $t=\frac{T}{4}$ is (where T is the time period) **[MP PET 1984]**

(a) $a\omega$ (b) $-a\omega$ (c) $a\omega^2$ (d) $-a\omega^2$

Solution : (d)

***Problem* 28.** A particle of mass m is hanging vertically by an ideal spring of force constant k, if the mass is made to oscillate vertically, its total energy is

(a) Maximum at extreme position (b) Maximum at mean position

(c) Minimum at mean position (d) Same at all position

Solution : (d)

15.11 Time Period and Frequency of S.H.M.

For S.H.M. restoring force is proportional to the displacement

$F \propto y$ or $F = -ky$ ...(i) where k is a force constant.

For S.H.M. acceleration of the body $A = -\omega^2 y$ ...(ii)

$\therefore$ Restoring force on the body $F = mA = -m\omega^2 y$ ...(iii)

From (i) and (iii) $ky = m\omega^2 y \Rightarrow \omega = \sqrt{\frac{k}{m}}$

$\therefore$ Time period $(T) = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

or Frequency $(n) = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

In different types of S.H.M. the quantities m and k will go on taking different forms and names.

In general m is called inertia factor and k is called spring factor.

Thus $T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

or $n = \frac{1}{2\pi}\sqrt{\frac{\text{Spring factor}}{\text{Inertia factor}}}$

In linear S.H.M. the spring factor stands for force per unit displacement and inertia factor for mass of the body executing S.H.M. and in Angular S.H.M. k stands for restoring torque per unit angular displacement and inertial factor for moment of inertia of the body executing S.H.M.

For linear S.H.M. $T = 2\pi\sqrt{\frac{m}{k}} = \sqrt{\frac{m}{\text{Force/Displacement}}} = 2\pi\sqrt{\frac{m \times \text{Displacement}}{m \times \text{Acceleration}}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{y}{A}}$

or $n = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Dispalcement}}} = \frac{1}{2\pi}\sqrt{\frac{A}{y}}$

15.12 Differential Equation of S.H.M.

For S.H.M. (linear) Acceleration $\propto$ – (Displacement)

$A \propto -y$

or $A = -\omega^2 y$

or $$\frac{d^2y}{dt^2} = -\omega^2 y$$

or $$m\frac{d^2y}{dt^2} + ky = 0 \qquad \left[\text{As } \omega = \sqrt{\frac{k}{m}}\right]$$

For angular S.H.M. $\tau = -c\theta$ and $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$

where $\omega^2 = \frac{c}{I}$ [As c = Restoring torque constant and I = Moment of inertia]

Sample problems based on Differential equation of S.H.M.

***Problem* 29.** A particle moves such that its acceleration a is given by $a = -bx$. Where x is the displacement from equilibrium position and b is a constant. The period of oscillation is

[NCERT 1984; CPMT 1991; MP PMT 1994; MNR 1995]

(a) $2\pi\sqrt{b}$ (b) $\frac{2\pi}{\sqrt{b}}$ (c) $\frac{2\pi}{b}$ (d) $2\sqrt{\frac{\pi}{b}}$

Solution : (b) We know that Acceleration = $-\omega^2$ (displacement) and $a = -bx$ (given in the problem)

Comparing above two equation $\omega^2 = b \Rightarrow \omega = \sqrt{b}$ $\therefore$ Time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{b}}$

***Problem* 30.** The equation of motion of a particle is $\frac{d^2y}{dt^2} + ky = 0$ where k is a positive constant. The time period of the motion is given by

(a) $\frac{2\pi}{k}$ (b) $2\pi k$ (c) $\frac{2\pi}{\sqrt{k}}$ (d) $2\pi\sqrt{k}$

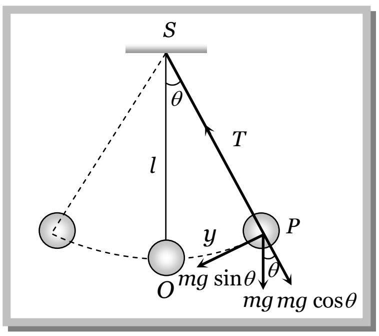
Solution : (c) Standard equation $m\frac{d^2y}{dt^2} + ky = 0$ and in a given equation m =1 and $k = k$

So, $T = 2\pi\sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{k}}$

15.13 Simple Pendulum

An ideal simple pendulum consists of a heavy point mass body suspended by a weightless, inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.

But in reality neither point mass nor weightless string exist, so we can never construct a simple pendulum strictly according to the definition.



Let mass of the bob = m

Length of simple pendulum = l

Displacement of mass from mean position (OP) = x

When the bob is displaced to position P, through a small angle θ from the vertical. Restoring force acting on the bob

$$F = -mg\sin\theta$$

or $$F = -mg\,\theta \qquad \left(\text{When } \theta \text{ is small } \sin\theta \simeq \theta = \frac{\text{Arc}}{\text{Length}} = \frac{OP}{l} = \frac{x}{l}\right)$$

or $F = -mg\dfrac{x}{l}$

$\therefore$ $\dfrac{F}{x} = \dfrac{-mg}{l} = k$ (Spring factor)

So time period $T = 2\pi\sqrt{\dfrac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\dfrac{m}{mg/l}} = 2\pi\sqrt{\dfrac{l}{g}}$

Important points

(i) The period of simple pendulum is independent of amplitude as long as its motion is simple harmonic. But if θ is not small, $\sin\theta \neq \theta$ then motion will not remain simple harmonic but will become oscillatory. In this situation if θ_0 is the amplitude of motion. Time period

$$T = 2\pi\sqrt{\frac{l}{g}}\left[1 + \frac{1}{2^2}\sin^2\left(\frac{\theta_0}{2}\right) + ...\right] \approx T_0\left[1 + \frac{{\theta_0}^2}{16}\right]$$

(ii) Time period of simple pendulum is also independent of mass of the bob. This is why

(a) If the solid bob is replaced by a hollow sphere of same radius but different mass, time period remains unchanged.

(b) If a girl is swinging in a swing and another sits with her, the time period remains unchanged.

(iii) Time period $T \propto \sqrt{l}$ where l is the distance between point of suspension and center of mass of bob and is called effective length.

(a) When a sitting girl on a swinging swing stands up, her center of mass will go up and so l and hence T will decrease.

(b) If a hole is made at the bottom of a hollow sphere full of water and water comes out slowly through the hole and time period is recorded till the sphere is empty, initially and finally the center of mass will be at the center of the sphere. However, as water drains off the sphere, the center of mass of the system will first move down and then will come up. Due to this l and hence T first increase, reaches a maximum and then decreases till it becomes equal to its initial value.

(iv) If the length of the pendulum is comparable to the radius of earth then $T = 2\pi\sqrt{\dfrac{1}{g\left[\dfrac{1}{l} + \dfrac{1}{R}\right]}}$

(a) If $l << R$, then $\dfrac{1}{l} >> \dfrac{1}{R}$ so $T = 2\pi\sqrt{\dfrac{l}{g}}$

(b) If $l >> R (\rightarrow \infty) 1/l < 1/R$ so $T = 2\pi\sqrt{\dfrac{R}{g}} = 2\pi\sqrt{\dfrac{6.4\times 10^6}{10}} \cong 84.6$ *minutes*

and it is the maximum time period which an oscillating simple pendulum can have

(c) If $l = R$ so $T = 2\pi\sqrt{\dfrac{R}{2g}} \cong 1 hour$

(v) If the bob of simple pendulum is suspended by a wire then effective length of pendulum will increase with the rise of temperature due to which the time period will increase.

$l = l_0(1 + \alpha\,\Delta\theta)$ (If $\Delta\theta$ is the rise in temperature, l_0 = initial length of wire, l = final length of wire)

$$\frac{T}{T_0} = \sqrt{\frac{l}{l_0}} = (1 + \alpha\,\Delta\theta)^{1/2} \approx 1 + \frac{1}{2}\alpha\,\Delta\theta$$

So $\quad \dfrac{T}{T_0} - 1 = \dfrac{1}{2}\alpha\,\Delta\theta \qquad$ *i.e.* $\quad \dfrac{\Delta T}{T} \approx \dfrac{1}{2}\alpha\,\Delta\theta$

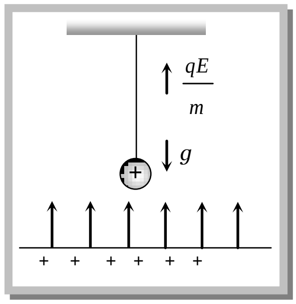
(vi) If bob a simple pendulum of density ρ is made to oscillate in some fluid of density σ (where $\sigma < \rho$) then time period of simple pendulum gets increased.

As thrust will oppose its weight therefore $\quad mg' = mg - \text{Thrust}$

or $\quad g' = g - \dfrac{V\sigma g}{V\rho} \qquad$ *i.e.* $\quad g' = g\left[1 - \dfrac{\sigma}{\rho}\right] \Rightarrow \dfrac{g'}{g} = \dfrac{\rho - \sigma}{\rho}$

$\therefore \quad \dfrac{T'}{T} = \sqrt{\dfrac{g}{g'}} = \sqrt{\dfrac{\rho}{\rho - \sigma}} > 1$

(vii) If a bob of mass m carries a positive charge q and pendulum is placed in a uniform electric field of strength E directed vertically upwards.



In given condition net down ward acceleration $g' = g - \dfrac{qE}{m}$

So $\quad T = 2\pi\sqrt{\dfrac{l}{g - \dfrac{qE}{m}}}$

If the direction of field is vertically downward then time period $T = 2\pi\sqrt{\dfrac{l}{g + \dfrac{qE}{m}}}$

(viii) Pendulum in a lift : If the pendulum is suspended from the ceiling of the lift.

(a) If the lift is at rest or moving down ward /up ward with constant velocity.

$T = 2\pi\sqrt{\dfrac{l}{g}} \qquad$ and $\qquad n = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}}$

(b) If the lift is moving up ward with constant acceleration a

$T = 2\pi\sqrt{\dfrac{l}{g + a}} \qquad$ and $\qquad n = \dfrac{1}{2\pi}\sqrt{\dfrac{g + a}{l}}$

Time period decreases and frequency increases

(c) If the lift is moving down ward with constant acceleration a

$T = 2\pi\sqrt{\dfrac{l}{g - a}} \qquad$ and $\qquad n = \dfrac{1}{2\pi}\sqrt{\dfrac{g - a}{l}}$

Time period increase and frequency decreases

(d) If the lift is moving down ward with acceleration $a = g$

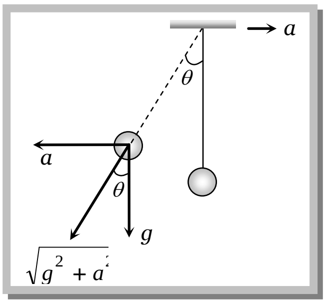
$T = 2\pi\sqrt{\dfrac{l}{g - g}} = \infty \qquad$ and $\qquad n = \dfrac{1}{2\pi}\sqrt{\dfrac{g - g}{l}} = 0$

It means there will be no oscillation in a pendulum.

Similar is the case in a satellite and at the centre of earth where effective acceleration becomes zero and pendulum will stop.

(ix) The time period of simple pendulum whose point of suspension moving horizontally with acceleration a

$$T = 2\pi\sqrt{\frac{l}{(g^2 + a^2)^{1/2}}} \quad \text{and} \quad \theta = \tan^{-1}(a/g)$$



(x) If simple pendulum suspended in a car that is moving with constant speed v around a circle of radius r.

$$T = 2\pi\frac{\sqrt{l}}{\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}}$$

(xi) Second's Pendulum : It is that simple pendulum whose time period of vibrations is two seconds.

Putting $T = 2\ sec$ and $g = 9.8 m/\sec^2$ in $T = 2\pi\sqrt{\frac{l}{g}}$ we get

$$l = \frac{4 \times 9.8}{4\pi^2} = 0.993\ m = 99.3\ cm$$

Hence length of second's pendulum is 99.3 *cm* or nearly 1 meter on earth surface.

For the moon the length of the second's pendulum will be 1/6 meter [As $g_{moon} = \frac{g_{\text{Earth}}}{6}$]

(xii) In the absence of resistive force the work done by a simple pendulum in one complete oscillation is zero.

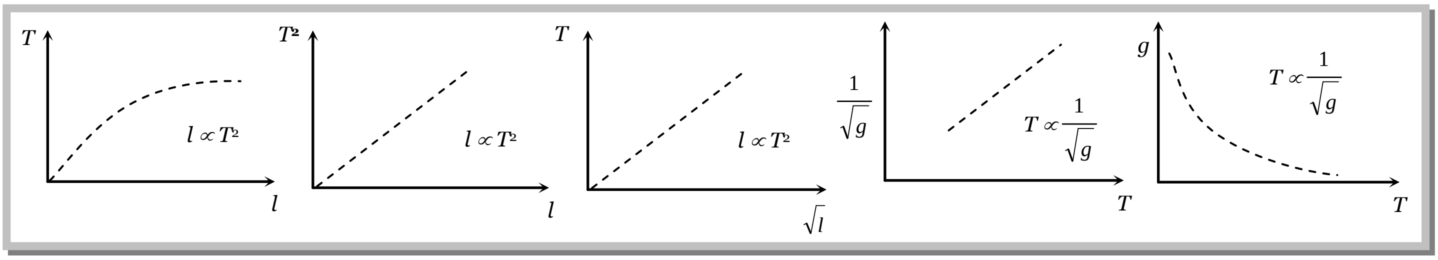
(xiii) Work done in giving an angular displacement θ to the pendulum from its mean position.

$$W = U = mgl(1 - \cos\theta)$$

(xiv) Kinetic energy of the bob at mean position = work done or potential energy at extreme

$$KE_{mean} = mgl(1 - \cos\theta)$$

(xv) Various graph for simple pendulum



***S**ample problems based on Simple pendulum*

***Problem* 31.** A clock which keeps correct time at 20°*C*, is subjected to 40°*C*. If coefficient of linear expansion of the pendulum is 12×10^{-6} / °*C*. How much will it gain or loose in time **[BHU 1998]**

(a) 10.3 *sec*/*day* (b) 20.6 *sec*/day (c) 5 *sec*/day (d) 20 *min*/day

Solution : (a) $\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta = \frac{1}{2} \times 12 \times 10^{-6} \times (40 - 20)$; $\Delta T = 12 \times 10^{-5} \times 86400$ *sec* / day $= 10.3$ *sec*/day.

***Problem* 32.** The metallic bob of simple pendulum has the relative density ρ. The time period of this pendulum is T. If the metallic bob is immersed in water, then the new time period is given by **[SCRA 1998]**

(a) $T\left(\frac{\rho - 1}{\rho}\right)$ (b) $T\left(\frac{\rho}{\rho - 1}\right)$ (c) $T\sqrt{\frac{\rho - 1}{\rho}}$ (d) $T\sqrt{\frac{\rho}{\rho - 1}}$

Solution : (d) Formula $\frac{T'}{T} = \sqrt{\frac{\rho}{\rho - \sigma}}$ Here $\sigma = 1$ for water so $T' = T\sqrt{\frac{\rho}{\rho - 1}}$.

***Problem* 33.** The period of a simple pendulum is doubled when **[CPMT 1974; MNR 1980; AFMC 1995]**

(a) Its length is doubled

(b) The mass of the bob is doubled

(c) Its length is made four times

(d) The mass of the bob and the length of the pendulum are doubled

Solution : (c)

***Problem* 34.** A simple pendulum is executing S.H.M. with a time period T. if the length of the pendulum is increased by 21% the percentage increase in the time period of the pendulum is **[BHU 1994]**

(a) 10% (b) 21% (c) 30% (d) 50%

Solution : (a) As $T \propto \sqrt{l}$ $\therefore$ $\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{1.21} \Rightarrow T_2 = 1.1T = T + 10\%T$.

***Problem* 35.** The length of simple pendulum is increased by 1% its time period will **[MP PET 1994]**

(a) Increase by 1% (b) Increase by 0.5% (c) Decrease by 0.5% (d) Increase by 2%

Solution : (b) $T = 2\pi\sqrt{l/g}$ hence $T \propto \sqrt{l}$

Percentage increment in $T = \frac{1}{2}$ (percentage increment in l) = 0.5%.

***Problem* 36.** The bob of a simple pendulum of mass m and total energy E will have maximum linear momentum equal to

[MP PMT 1986]

(a) $\sqrt{\frac{2E}{m}}$ (b) $\sqrt{2mE}$ (c) $2mE$ (d) mE^2

Solution : (b) $E = \frac{P^2}{2m}$ where E = Kinetic Energy, P = Momentum, m = Mass

So $P = \sqrt{2mE}$.

***Problem* 37.** The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (if it is a second's pendulum on earth) **[IIT 1973]**

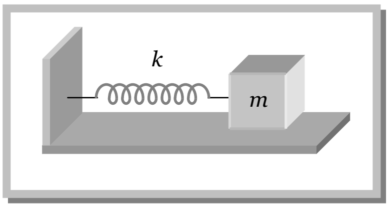
(a) $\frac{1}{\sqrt{2}}$ *sec* (b) $2\sqrt{2}$ *sec* (c) 2 *sec* (d) $\frac{1}{2}$ *sec*

Solution : (b) $g \propto \frac{M}{R^2}$; $g' = g/2$; $\frac{T'}{T} = \sqrt{\frac{g}{g'}}$ (T = 2 sec for second's pendulum)

$T' = 2\sqrt{2}$

15.14 Spring Pendulum

A point mass suspended from a mass less spring or placed on a frictionless horizontal plane attached with spring (fig.) constitutes a linear harmonic spring pendulum



Time period $T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$

$T = 2\pi\sqrt{\frac{m}{k}}$ and Frequency $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Important points

(i) Time period of a spring pendulum depends on the mass suspended

$$T \propto \sqrt{m} \qquad \text{or} \qquad n \propto \frac{1}{\sqrt{m}}$$

i.e. greater the mass greater will be the inertia and so lesser will be the frequency of oscillation and greater will be the time period.

(ii) The time period depends on the force constant k of the spring

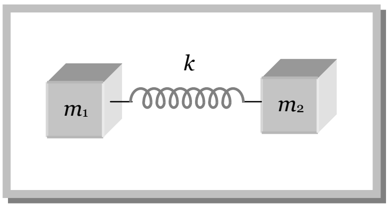
$$T \propto \frac{1}{\sqrt{k}} \qquad \text{or} \qquad n \propto \sqrt{k}$$

(iii) Time of a spring pendulum is independent of acceleration due to gravity. That is why a clock based on spring pendulum will keep proper time every where on a hill or moon or in a satellite and time period of a spring pendulum will not change inside a liquid if damping effects are neglected.

(iv) If the spring has a mass M and mass m is suspended from it, effective mass is given by $m_{eff} = m + \frac{M}{3}$

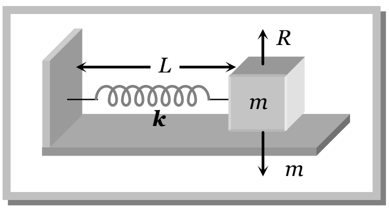
So that $T = 2\pi\sqrt{\frac{m_{eff}}{k}}$

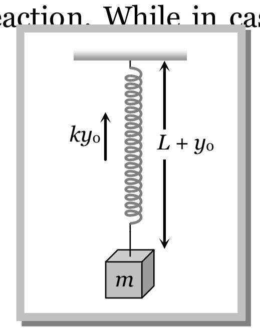
(v) If two masses of mass m_1 and m_2 are connected by a spring and made to oscillate on horizontal surface, the reduced mass m_r is given by $\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2}$



So that $T = 2\pi\sqrt{\frac{m_r}{k}}$

(vi) If a spring pendulum, oscillating in a vertical plane is made to oscillate on a horizontal surface, (or on inclined plane) time period will remain unchanged. However, equilibrium position for a spring in a horizontal plain is the position of natural length of spring as weight is balanced by reaction. While in case of vertical motion equilibrium position will be $L + y_0$ with $ky_0 = mg$



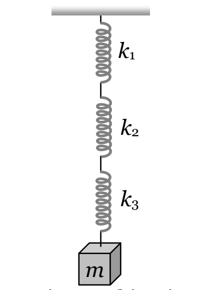


(vii) If the stretch in a vertically loaded spring is y_0 then for equilibrium of mass m, $ky_0 = mg$ *i.e.* $\frac{m}{k} = \frac{y_0}{g}$

So that $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$

Time period does not depends on 'g' because along with g, y_0 will also change in such a way that $\frac{y_0}{g} = \frac{m}{k}$ remains constant

(viii) Series combination : If n springs of different force constant are connected in series having force constant $k_1, k_2, k_3 ....$ respectively then



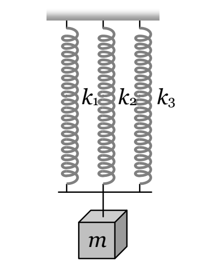
Series combination

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ...$$

If all spring have same spring constant then

$$k_{eff} = \frac{k}{n}$$

(ix) Parallel combination : If the springs are connected in parallel then



Parallel combination

$$k_{eff} = k_1 + k_2 + k_3 + ...$$

If all spring have same spring constant then

$$k_{eff} = nk$$

(x) If the spring of force constant k is divided in to n equal parts then spring constant of each part will become nk and if these n parts connected in parallel then

$$k_{eff} = n^2 k$$

(xi) The spring constant k is inversely proportional to the spring length.

As $k \propto \frac{1}{\text{Extension}} \propto \frac{1}{\text{Length of spring}}$

That means if the length of spring is halved then its force constant becomes double.

(xii) When a spring of length l is cut in two pieces of length l_1 and l_2 such that $l_1 = nl_2$.

If the constant of a spring is k then Spring constant of first part $k_1 = \frac{k(n+1)}{n}$

Spring constant of second part $k_2 = (n+1)k$

and ratio of spring constant $\frac{k_1}{k_2} = \frac{1}{n}$

Sample problems based on Spring pendulum

***Problem* 38.** A spring of force constant k is cut into two pieces such that one pieces is double the length of the other. Then the long piece will have a force constant of **[IIT-JEE 1999]**

(a) $2/3k$ (b) $3/2k$ (c) $3k$ (d) $6k$

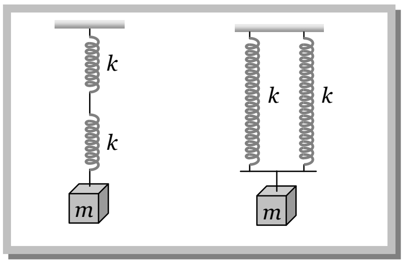
Solution : (b) If $l_1 = nl_2$ then $k_1 = \frac{(n+1)k}{n} = \frac{3}{2}k$ [As n = 2]

Problem 39. Two bodies M and N of equal masses are suspended from two separate mass less springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of M to that of N is

(a) k_1 / k_2 (b) $\sqrt{k_1 / k_2}$ (c) k_2 / k_1 (d) $\sqrt{k_2 / k_1}$

Solution : (d) Given that maximum velocities are equal $a_1\omega_1 = a_2\omega_2 \Rightarrow a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$.

Problem 40. Two identical springs of constant k are connected in series and parallel as shown in figure. A mass m is suspended from them. The ratio of their frequencies of vertical oscillation will be



(a) 2 : 1

(b) 1 : 1

(c) 1 : 2

(d) 4 : 1

Solution : (c) For series combination $n_1 \propto \sqrt{k/2}$

For parallel combination $n_2 \propto \sqrt{2k}$ so $\frac{n_1}{n_2} = \sqrt{\frac{k/2}{2k}} = \frac{1}{2}$.

Problem 41. A block of mass m attached to a spring of spring constant k oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. The block has a speed v when the spring is at its natural length. Before coming to an instantaneous rest, if the block moves a distance x from the Mean position, then

(a) $x = \sqrt{m/k}$ (b) $x = \frac{1}{v}\sqrt{\frac{m}{k}}$ (c) $x = v\sqrt{m/k}$ (d) $x = \sqrt{mv/k}$

Solution : (c) Kinetic energy of block $\left(\frac{1}{2}mv^2\right)$ = Elastic potential energy of spring $\left(\frac{1}{2}kx^2\right)$

By solving we get $x = v\sqrt{\frac{m}{k}}$.

Problem 42. A block is placed on a friction less horizontal table. The mass of the block is m and springs of force constant k_1, k_2 are attached on either side with if the block is displaced a little and left to oscillate, then the angular frequency of oscillation will be

(a) $\left(\frac{k_1 + k_2}{m}\right)^{1/2}$ (b) $\left[\frac{k_1 k_2}{m(k_1 + k_2)}\right]^{1/2}$ (c) $\left[\frac{k_1 k_2}{(k_1 - k_2)m}\right]^{1/2}$ (d) $\left[\frac{k_1^2 + k_2^2}{(k_1 + k_2)m}\right]^{1/2}$

Solution : (a) Given condition match with parallel combination so $k_{eff} = k_1 + k_2 \quad \therefore \omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$.

Problem 43. A particle of mass 200 gm executes S.H.M. The restoring force is provided by a spring of force constant 80 N/m. The time period of oscillations is

(a) 0.31 sec (b) 0.15 sec (c) 0.05 sec (d) 0.02 sec

Solution : (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2}{80}} = \frac{2\pi}{20} = 0.31\,\text{sec}$.

Problem 44. The length of a spring is l and its force constant is k when a weight w is suspended from it. Its length increases by x. if the spring is cut into two equal parts and put in parallel and the same weight W is suspended from them, then the extension will be

(a) $2x$ (b) x (c) $x/2$ (d) $x/4$

Solution : (d) As $F = kx$ so $x \propto \frac{1}{k}$ (if F = constant)

If the spring of constant k is divided in to two equal parts then each parts will have a force constant $2k$.

If these two parts are put in parallel then force constant of combination will becomes $4k$.

$$x \propto \frac{1}{k} \qquad \text{so, } \frac{x_2}{x_1} = \frac{k_1}{k_2} = \frac{k}{4k} \Rightarrow x_2 = \frac{x}{4}.$$

***Problem* 45.** A mass m is suspended from a string of length l and force constant k. The frequency of vibration of the mass is f_1. The spring is cut in to two equal parts and the same mass is suspended from one of the parts. The new frequency of vibration of mass is f_2. Which of the following reaction between the frequencies is correct.

[NCERT 1983; CPMT 1986; MP PMT 1991]

(a) $f_1 = \sqrt{2} f_2$ (b) $f_1 = f_2$ (c) $f_1 = 2f_2$ (d) $f_2 = \sqrt{2} f_1$

Solution : (d) $f \propto \sqrt{k}$

If the spring is divided in to equal parts then force constant of each part will becomes double

$$\frac{f_2}{f_1} = \sqrt{\frac{k_2}{k_1}} = \sqrt{2} \Rightarrow f_2 = \sqrt{2} f_1$$

15.15 Various Formulae of S.H.M.

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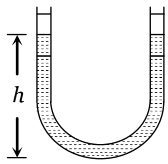
S.H.M. of a liquid in U tube

If a liquid of density ρ contained in a vertical U tube performs S.H.M. in its two limbs. Then time period $T = 2\pi\sqrt{\frac{L}{2g}} = 2\pi\sqrt{\frac{h}{g}}$

where L = Total length of liquid column,

h = Height of undisturbed liquid in each limb (L=2h)



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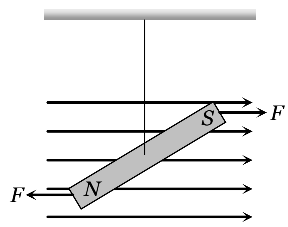
S.H.M. of a bar magnet in a magnetic field

$T = 2\pi\sqrt{\frac{I}{MB}}$

I = Moment of inertia of magnet

M = Magnetic moment of magnet

B = Magnetic field intensity



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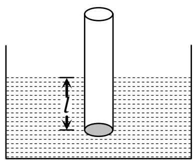
S.H.M. of a floating cylinder

If l is the length of cylinder dipping in liquid then time period $T = 2\pi\sqrt{\frac{l}{g}}$



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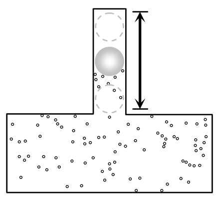
S.H.M. of ball in the neck of an air chamber

$T = \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$



m = mass of the ball

V = volume of air- chamber

A = area of cross section of neck

E = Bulk modulus for Air

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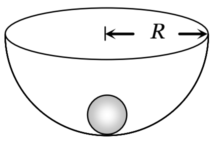
S.H.M. of a small ball rolling down in hemi-spherical bowl

$T = 2\pi\sqrt{\frac{R-r}{g}}$



R = radius of the bowl

r =radius of the ball

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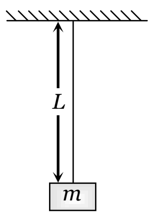
S.H.M. of a body suspended from a wire

$T = 2\pi\sqrt{\frac{mL}{YA}}$



m = mass of the body

L = length of the wire

Y = young's modulus of wire

A = area of cross section of wire

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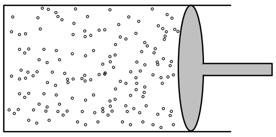
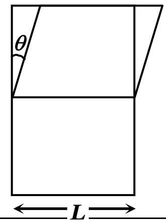
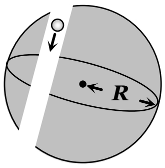
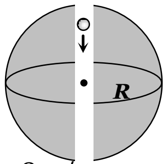
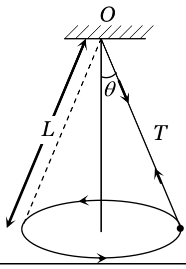
S.H.M. of a piston in a cylinder	**S.H.M of a cubical block**
$T = 2\pi\sqrt{\frac{Mh}{PA}}$ M = mass of the piston A = area of cross section h = height of cylinder P = pressure in a cylinder 	$T = 2\pi\sqrt{\frac{M}{\eta L}}$ M = mass of the block L = length of side of cube η = modulus of rigidity 
S.H.M. of a body in a tunnel dug along any chord of earth	**S.H.M. of body in the tunnel dug along the diameter of earth**
$T = 2\pi\sqrt{\frac{R}{g}} = 84.6$ minutes  	$T = 2\pi\sqrt{\frac{R}{g}}$ T = 84.6 minutes R = radius of the earth = 6400km g = acceleration due to gravity = 9.8m/s^2 at earth's surface 
S.H.M. of a conical pendulum	**S.H.M. of *L-C* circuit**
$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$ L = length of string θ = angle of string from the vertical g = acceleration due to gravity  	$T = 2\pi\sqrt{LC}$ L = coefficient of self inductance C = capacity of condenser

15.16 Important Facts and Formulae

(1) When a body is suspended from two light springs separately. The time period of vertical oscillations are T_1 and T_2 respectively.

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}} \quad \therefore \quad k_1 = \frac{4\pi^2 m}{T_1^{\;2}} \text{ and } \quad T_2 = 2\pi\sqrt{\frac{m}{k_2}} \quad \therefore \; k_2 = \frac{4\pi^2 m}{T_2^{\;2}}$$

When these two springs are connected in series and the same mass m is attached at lower end and then for series combination $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

By substituting the values of k_1, k_2 $\quad \frac{T^2}{4\pi^2 m} = \frac{T_1^2}{4\pi^2 m} + \frac{T_2^2}{4\pi^2 m}$

Time period of the system $T = \sqrt{T_1^{\;2} + T_2^{\;2}}$

When these two springs are connected in parallel and the same mass m is attached at lower end and then for parallel combination $k = k_1 + k_2$

By substituting the values of k_1, k_2 $\quad \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 m}{T_1^{\;2}} + \frac{4\pi^2 m}{T_2^{\;2}}$

Time period of the system $T = \frac{T_1 T_2}{\sqrt{T_1^{\;2} + T_2^{\;2}}}$

(2) The pendulum clock runs slow due to increase in its time period whereas it becomes fast due to decrease in time period.

(3) If infinite spring with force constant $k, 2k, 4k, 8k$.......... respectively are connected in series. The effective force constant of the spring will be $k/2$.

(4) If $y_1 = a\sin\omega t$ and $y_2 = b\cos\omega t$ are two S.H.M. then by the superimposition of these two S.H.M. we get

$$\vec{y} = \vec{y}_1 + \vec{y}_2$$

$$y = a\sin\omega t + b\cos\omega t$$

$$y = A\sin(\omega t + \phi) \quad \text{this is also the equation of S.H.M.}$$

where $A = \sqrt{a^2 + b^2}$ and $\phi = \tan^{-1}(b/a)$

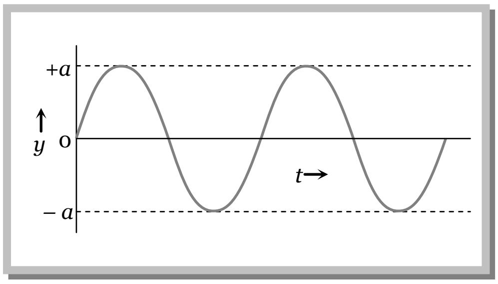
(5) If a particle performs S.H.M. whose velocity is v_1 at a x_1 distance from mean position and velocity v_2 at distance x_2

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}\ ; \quad T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}} \quad a = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}\ ; \quad v_{\max} = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{x_2^2 - x_1^2}}$$

15.17 Free, Damped, Forced and Maintained Oscillation

(1) **Free oscillation**

(i) The oscillation of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations

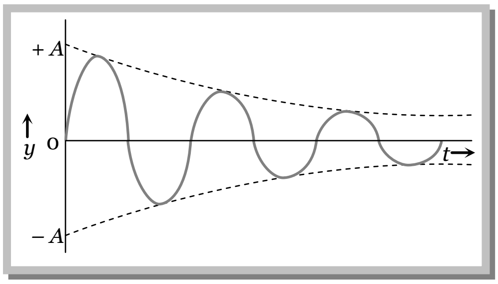


(ii) The amplitude, frequency and energy of oscillation remains constant

(iii) Frequency of free oscillation is called natural frequency because it depends upon the nature and structure of the body.

(2) **Damped oscillation**

(i) The oscillation of a body whose amplitude goes on decreasing with time are defined as damped oscillation



(ii) In these oscillation the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, hysteresis *etc*.

(iii) Due to decrease in amplitude the energy of the oscillator also goes on decreasing exponentially

(3) **Forced oscillation**

(i) The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation

(ii) The amplitude of oscillator decrease due to damping forces but on account of the energy gained from the external source it remains constant.

(iii) Resonance : When the frequency of external force is equal to the natural frequency of the oscillator. Then this state is known as the state of resonance. And this frequency is known as resonant frequency.

(4) **Maintained oscillation**

The oscillation in which the loss of oscillator is compensated by the supplying energy from an external source are known as maintained oscillation.

Introduction :

Sound is always produced by some vibrating body. The vibrating body generates mechanical waves and these waves spreads in the surrounding medium. We are aware that these waves propagate in the form of a series of compressions and rarefactions in air or the surrounding medium. When reached upto the human ear drum it causes a sensation of hearing. As far as architectural acoustics are concerned, we are interested the combined effect of sound waves which creates a sense of sound on human ear.

Some important characteristics

(1) The propagation of sound requires the presence of an elastic medium.

(2) Sound can not travel through vacuum

(3) The compression and rarefactions due to a sound modulate the normal atmospheric pressure with small pressure changes occuring regularly above and below it.

(4) The velocity of sound depends on the nature and temperature of the medium.

1.1 Classification of Sound :

Based upon frequency of sound waves, it can be classified into its three main categories.

(a) Audible waves : Sound waves with frequency in the range of 20Hz to 20KHz.

(b) Infrasonic waves : Sound waves below audible range i.e. below 20Hz.

(c) Ultrasonic waves : Sound waves above audible range i.e. 20KHz.

1.1.1 Characteristics of Musical Sound :

❋❋❋ **[University Exam : June 2009 !!!]**

Musical sounds are distinguished from noises in that they are composed of regular, uniform vibrations, while noises are irregular and disordered vibrations. One musical tone is distinguished from another on the basis of pitch, intensity, or loudness and quality, or timbre. Pitch describes how high or low a tone is and depends upon the rapidity with which a sounding body vibrates, i.e. upon the frequency of vibration. The higher the frequency of vibration, the higher the tone; the pitch of a siren gets higher and higher as the frequency of vibration increases. The apparent change in the pitch of a sound as a source approaches or moves away from an observer is described by the Doppler effect. The intensity or loudness of a sound depends upon the extent to which the sounding body vibrates, i.e. the amplitude of vibration. A sound is louder as the amplitude of vibration is greater, and the intensity decreases as the distance from the source increases. Loudness is measured in units called decibels. The sound waves given off by different vibrating bodies differ in quality, or timbre. A note from a saxophone, for instance, differs from a note of the same pitch and intensity produced by a violin or a xylophone; similarly vibrating reeds, columns of air, and strings all differ. Quality is dependent on the number and relative intensity of overtones produced by the vibrating body (see harmonic), and these in turn depend upon the nature of the vibrating body.

1.2 Important Terms Used :

In the study of sound waves we come across various terms like Pitch (*This law does not hold good near the upper and lower limits of audiability*), Timber which basically deals with the quality of the sound waves and source. At the same time for technical assessment, we make use of important parameters like intensity and loudness.

1.2.1 Weber Fechner Law :

✱✱✱ [University Exam : Dec. 2008 !!!]

This law has it roots hidden in psychology and proved scientifically according to which : The loudness of sound sensed by ear is directly proportional to logarithm of its intensity.

∴ According to Weber-Fechner law :

Suppose the loudness is S for intensity I and S_0 for intensity I_0,

$$\therefore \quad S = K \log_{10} I$$

$$S_0 = K \log_{10} I_0$$

The intensity level L is the difference in loudness.

$$L = S - S_0$$

$$= K \log_{10} I - K \log_{10} I_0 = K \log_{10} \left(\frac{I}{I_0}\right)$$

take, $K = 1$

$$L = \log_{10} \left(\frac{I}{I_0}\right) \qquad \ldots(1.1)$$

Intensity and loudness are the two words which are similar but with slight difference.

Table 1.1

Sr. No.	Intensity	Loudness
1.	Defined as the quantity of energy propagating through a unit area per unit time, in the direction of propagation being perpendicular to the area (unit : watt/m^2).	It is just an aural sensation and it a physiological phenomenon rather than a physical one.
2.	It refers to the external or the objective measurement.	It refers to an internal or subjective aspect.
3.	It is a physical quantity.	Merely a degree of sensation.

Loudness 'S' increases with intensity 'I' as per the following relation*

or $$S \;\alpha\; \log I \qquad \ldots(1.2)$$

$$\frac{dS}{dI} = \frac{K}{I} \qquad \ldots(1.3)$$

Where K is proportionality constant

Here $\frac{ds}{dI}$ is called the **sensitiveness** of the ear.

In practice, it is the relative intensity that is important and not the absolute value. Hence the intensity of sound is often measured as the ratio to a standard intensity I_0. The intensity level is I / I_0.

- The standard intensity taken is $I_0 = 10^{-12}$ watts / m^2. (It is an arbitrarily selected value. It is an intensity that can just be heard at frequency 1 kHz)

1.2.2 Bel :

- As discussed in art 1.2.1, whenever the intensity of sound increases by a factor of 10, the increase in the intensity is said to be 1 bel *(A unit named after Alexander Graham Bell, the inventor of telephone)*
- Therefore dynamic range of audibility of the human ear is 12 bels or 120 dB. When the intensity increases by a factor of $10^{0.1}$, the increase in intensity is 0.1 bel or 1dB.

From Equation 1.1

$$L = \log_{10}\left(\frac{I}{I_0}\right)$$

$\therefore$ in decibel

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

For the intensity level change = 1 dB

$$1 = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

$$\therefore \frac{I}{I_0} = 1.26 \qquad \ldots(1.4)$$

If $\quad I = I_0,$

$$L = 10 \log 1 = 0$$

This represents the threshold of audibility.

It means that intensity level alters by 1dB when intensity of sound changes by 26%

Table 1.2 : Intensity levels of different sounds

Sr. No.	Sound	Intensity level (in db)
(1)	Threshold of hearing	0
(2)	Rustle of leaves	10
(3)	Whisper	15 – 20
(4)	Normal conversation	60 – 65
(5)	Heavy traffic	70 – 80
(6)	Thunder	100 – 110
(7)	Painful sound	130 and above

1.2.3 Phon :

- The intensity levels given in the above Table 1.2 refer to the loudness in decibels with the assumption that the threshold of audibility is the same irrespective of the pitch ***(Pitch is** a subjective sensation perceived when a tone of a given frequency is sounded. It enables us to classify a note as high or low and to distinguish a shrill sound from a flat sound of the same intensity on the same instrument.)* of the sound.
- However, the sensitivity of the ear and the threshold audibility vary over wide ranges of frequency and intensity.
- Hence the intensity level will be different at different frequencies even for the same value of I_0.
- For measuring the intensity level a different unit called **phon** is used.
- The measure of loudness in phons of any sound is equal to the intensity level in decibels of an equally loud pure tone of frequency 1000 Hz.
- Hence Phon scale and decibel scale agree for a frequency of 1000 Hz but the two values differ at other frequencies.
- Suppose the intensity level of a note of frequency 480 Hz is to be determined. A standard source of frequency 1000 Hz is sounded and the intensity of the standard source is adjusted so that it is equal to the loudness of the given note of frequency 480 Hz.
- The intensity level of the standard source in decibels is numerically equal to the loudness of the given source in phons.

Ex. 1.1 : Calculate the change in intensity level when the intensity of sound increases 100 times its original intensity.

Soln. :

Given :

$$\text{Initial intensity} = I_0$$

$$\text{Final intensity} = I$$

$$\frac{I}{I_0} = 100$$

$$\text{Increase in intensity level} = L$$

$$\therefore \quad L = 10 \log_{10}\left(\frac{I}{I_0}\right) \text{ (in dB)}$$

$$\therefore \quad \mathbf{L = 10 \log_{10} 100 = 20 \ dB} \qquad \textbf{...Ans.}$$

Ex. 1.2 : Find the intensity level in phons if 3000 Hz with intensity level of 70 dB produces the same loudness as a standard source of frequency 1000 Hz at a intensity level 67 dB.

Soln. :

As the 3000 Hz source has the same loudness of standard source of 1000 Hz with 67 db, the intensity level of the note of frequency **3000 Hz is 67 phons.** **…Ans.**

1.3 Architectural Acoustics :

Lets try to understand what exactly acoustics of a hall means. Consider the following cases :

(a) Imagine a hall, it is easy for any one to understand that sound produced at a point will reach the other point directly as well as after reflections from walls, roof etc. The intensity of the sound depends on the distance covered by sound on different paths. These sounds are generally out of phase and due to interference the distribution of intensity in the room is not uniform.

(b) It is also important to consider a possibility that the different frequency sounds of a musical instrument may interfere differently at some point and quality of music may become unpleasent.

(c) It is known that sound persists for some time due to multiple reflections, even when the original sound has ceased. During this time if any other syllable is received, superimposition of these two will affect audiability as both will remain indistinct. If this takes place during a speech, a confusion will be created.

(d) Concentration of sound taking place at any part of the hall.

The above mentioned points are very common but needs a special scientific attention. **Prof. W.C. Sabine** was the first person who took it seriously.

1.4 Reverberation Time :

❋❋❋ [University Exam : Dec. 2008 !!!]

Reverberation means the prolonged reflection of sound from walls, floor or roof of a hall. In simple language it is nothing but **persistence of sound even after the sources of the sound has stopped**.

Reverberation time :

The time gap between the initial direct note and the reflected note upto a minimum audibility level is called reverberation time.

- More precisely, the interval of time taken by a sustained or continuous sound to fall to an intensity level equal to one millionth of its original value. (i.e. fall by 60 db in loudness) is called reverberation time.
- In a good auditorium it is necessary to keep the reverberation time as small as possible. The intensity of the sound as received by listener is shown graphically in Fig. 1.1.

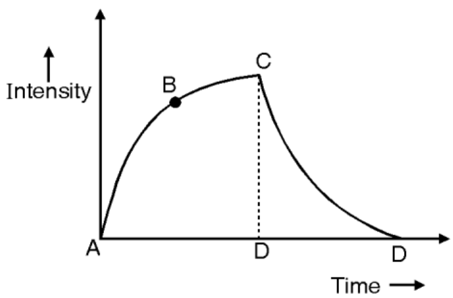


Fig. 1.1

- When a source emits sound, the waves spread out and the listener is aware of the commencement of sound when the direct waves reach his ears. Subsequently the listener receives sound energy due to reflected waves also. If the note is continuously sounded, the intensity of sound at the listener's ears gradually increases. After sometime, a balance is reached between the energy emitted per second by the source and energy lost or dissipated by walls or other materials.
- The resultant energy attains an average steady value and to the listener the intensity of sound appears to be steady and constant.
- This is represented by a portion BC of the curve ABCD.
- If at C, the source stops emitting sound, the intensity of sound falls exponentially as shown by the curve CD.

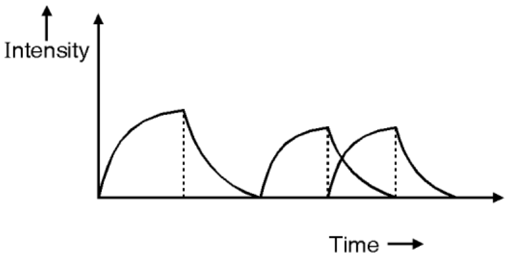


Fig. 1.2

- When intensity of sound falls below the minimum audibility level, the listener will not get the sound.
- When a series of notes are produced in an auditorium each note will give rise to its own intensity curve with respect to time. The curve for these notes are shown in Fig. 1.2.

  In order to maintain distinctness in speech it is necessary that :

  (a) Each separate note should give sufficient intensity of sound in every part of the auditorium.

  (b) Each note should die down rapidly before the maximum average intensity due to the next note is heared by the listener.

1.5 Absorption :

When a sound wave strikes a surface there are three possibilities.

(a) Part of energy is absorbed

(b) Part of it is transmitted

(c) Remaining energy is reflected

- The effectiveness of surface in absorbing sound energy is expressed by absorption coefficient denoted by **a.**

$$a = \frac{\text{Sound energy absorbed by the surface}}{\text{Total sound energy incident on the surface}} \qquad \dots(1.5)$$

- For the comparison of relative efficiencies of different absorbing material, it is necessary to select a standard or reference.
- Sabine selected a unit area of open window, as standard. For any open window the sound falling on it completely passes out no reflection, and more importantly no absorption.

- Hence open window is an ideal absorber of the sound. The absorption coefficient is measured in open window unit.

(OWU) or Sabine :

- The absorption coefficient of a material is defined as the reciprocal of its area which absorbs the same sound energy as absorbed by unit area of open window.
- Effective absorbing area A of the surface having total area S and absorption coefficient 'a' is given by

$$A = a\,S \qquad \ldots(1.6)$$

- If the $a_1, a_2, a_3, \ldots, a_n$ are the absorption coefficients for each reflecting surface and $S_1, S_2, S_3, \ldots S_n$ are the corresponding areas, then the average value of absorption co-efficient is

$$a + 89 = \frac{a_1 S_1 + a_2 S_2 + a_3 S_3 + \ldots\ldots + a_n S_n}{S_1 + S_2 + S_3 + \ldots + S_n}$$

$$= \frac{\sum_{i=1}^{n} a_i S_i}{S} \qquad \ldots(1.7)$$

Where S is total surface area.

1.6 Sabine's Formula :

✱✱✱ **[University Exam : Dec. 2008 !!!]**

- Prof. W.C. Sabine observed the concept of reverberation time for varieties of conditions like empty room, furnished room, small room, auditorium etc.
- He concluded the following,
  (a) Reverberation time depends upon reflectivity of sound form various surfaces available in side the hall. If the reflection is good, reverberation time of the hall will be longer as sound take more time to die out.
  (b) Reverberation time depends upon volume of the hall.
  i.e. $T \propto V$
  (c) Reverberation time depends upon coefficient of absorption of various surfaces present in the hall. For shorter reverberation, absorption should be more.
  (d) As absorption coefficient is found to be increased with increase in frequency, reverberation time decreases with frequency.

$\therefore$ Reverberation time $\quad T \propto \dfrac{V}{A}$

where, V = Volume of hall

A = Absorption

or $\quad T = K\dfrac{V}{A}$

where, K = Proportionality constant

- It has been further observed that is all the parameters are taken in SI then, proportionality constant is found to be 0.161.

$$\therefore \quad T = 0.161 \frac{V}{A} \qquad \ldots(1.8)$$

Equation (1.8) is Sabine's formula.

Absorption A given in Equation (1.8) represents overall absorption which is given as

$$A = \sum_{i=1}^{n} a\,S = a_1 S_1 + a_2 S_2 + \ldots\ldots + a_n S_n$$

Ex. 1.3 : For an empty assembly hall of size 20 × 15 × 10 cubic meter with absorption coefficient 0.106 . Calculate reverberation time.

Soln. :

Given :

(i) Size of the room $= 20 \times 15 \times 10$

$= 3000$ cubic meter

(ii) $a = 0.106$

Formula $T = 0.161 \frac{V}{A}$

$= 0.161 \frac{V}{\sum aS}$

Here S = Total surface area of the hall is given by $2\,(20 \times 15 + 15 \times 10 + 20 \times 10)$

$= 1300$ sqm

$\therefore$ Reverberation time $T = 0.161 \times \frac{3000}{0.106 \times 1300}$

$\therefore$ Reverberation time = 3.5 sec **...Ans.**

1.7 Determination of Absorption Coefficient :

Step 1 : Using a source of sound inside the hall, reverberation time is measured with the help of chronograph without inserting any test material (whose co-efficient of absorption is to be calculated). Let the reverberation time be T_1,

$$\therefore \quad T_1 = 0.161 \frac{V}{A}$$

$$= 0.161 \frac{V}{\sum aS}$$

$$\therefore \quad \frac{1}{T_1} = \frac{\sum aS}{0.161\,V} \qquad \ldots(1.9)$$

Step 2 : Now consider a material like curtain or stage screen whose co-efficient of absorption is to be found out suspended inside the room and reverberation time T_2 is obtained. **Since the material is suspended in hall, surface area from both the side are to be considered.**

$$\therefore \quad \frac{1}{T_2} = \frac{0.161\,V}{\sum aS + 2a_2 S_2}$$

where a_2 = Co-efficient of absorption of the material under investigation

S_2 = Surface of the material (since both the sides are used, it is multiplied by 2)

$$\therefore \quad \frac{1}{T_2} = \frac{\sum aS + 2a_2\, S_2}{0.161\ V} \qquad \ldots(1.10)$$

From Equation (1.9) and (1.10)

$$\frac{1}{T_2} - \frac{1}{T_1} = \frac{1}{0.161} \cdot \frac{2a_2\, S_2}{V}$$

$$\therefore \quad 2a_2\, S_2 = 0.161\ V \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\therefore \quad a_2 = \frac{0.161\ V}{2\ S_2} \left(\frac{1}{T_2} - \frac{1}{T_1}\right) \qquad \ldots(1.11)$$

All the quantities on RHS are known, co-efficient of absorption of an absorbing material which is suspended in hall with both the surfaces open can be calculated.

Table 1.3 : Absorption coefficients of some materials

Material	Absorption coefficient per m^2 at 500 Hz
Open window	1.0
Stage curtain	0.2
Common plaster	0.3
Carpet	0.4
Heavy curtain	0.5
Perforated cellulose fiber tiles	0.85

1.8 Conditions for Good Acoustic :

As already introduced in art 1.3, a lecture hall or auditorium should satisfy the following conditions in order to be acoustically good.

(a) The initial sound from the source should be of adequate intensity.

(b) The sound should spread evenly with proper loudness every where is the hall

(c) The sound of speech or music should be clear and words of or musical notes must be distinctly audible to all.

(d) All undesired or extraneous noise must be reduced to the extent that it will not interfere with normal hearing of speech or hearing.

(e) Any distortion due to shape and size must be absent.

1.9 Methods of Design for Good Acoustics :

✱✱✱ [University Exam : Dec. 2008, June 2009 !!!]

In order to make acoustically correct hall following points may be considered. These are merely the guidelines, depending upon specific requirement a justified step be taken.

(a) Selection of proper site :

Avoid noisy places like railway tack, roads with heavy traffic, airports, industrial vicinity for auditorium.

(b) Volume :

- Size of the hall/ auditorium should be such that it remains optimum.
- Small halls leads to irregular distribution of sound because of formation of standing waves.
- Too big halls may also create a weaker intensity and larger reverberation time which is a very serious issue.

(c) Shape :

- It is one of the most important parameter to be considered for acoustically correct hall.
- As the reflections are created by roof and side walls, they should be designed in such a way that echos are not allowed to generate.
- In place of parallel walls, splayed side walls are preferred. Curved surface on walls, ceilings or floor produce concentration of sound into particular region and absence of sound in other regions.
- **Hence curved surface must be designed with proper care.**

(d) Use of absorbents :

- Once the construction of hall is completed certain errors are found or the hall requires further correction as far as acoustics are concerned. For this use of absorbents is very common.
- As the reflections from rear wall are of no use. It must be covered with absorbents, so as the ceiling.
- False ceiling provided in large halls solves this problem effectively. The floor needs to be covered with carpet so as unwanted reflections and the noise created by audience is suppressed.

(e) Reverberation :

- Reverberation time must be maintained in such a that it does not remain too short or too large i.e. nearly 0.5 seconds for lecture hall, around 1.2 for concerts hall and around 2 for cinema halls.
- Proper use of absorbing materials, sufficient people as audience, presence of open windows presence of furniture etc are the major components which can decide the reverberation time.
- **Calculated use of such components will be helpful to either increase or decrease the reverberation time.**

(f) Echelon effect :

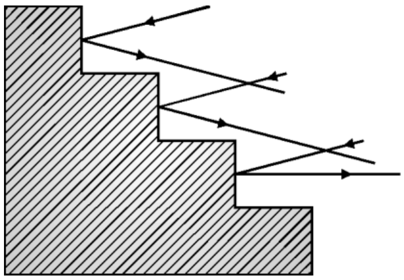


Fig. 1.3 : Echelon effect

- A set of railings or staircase or any regular spacing of reflected surfaces may produce a musical note due to regular succession of echoes of the original sound to listener.
- This makes original sound to appear confused. Either one should avoid use of such surfaces or keep them covered with thick carpet.

1.10 Solved Problems :

Ex. 1.4 : Calculate the change in intensity level when intensity level increases by 10^6 times its original intensity.

Soln. :

Given :

Initial intensity $= I_0$

Final intensity $= I$

$$\frac{I}{I_0} = 10^6$$

Increase in intensity level in dB

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right) = 10 \log_{10} (10^6)$$

$$\therefore \quad L = 60 \text{ dB}$$ **…Ans.**

Ex. 1.5 : A room has dimensions 6 × 4 × 5 meters calculate :

(a) the mean free path of the sound waves in the room

(b) the number of reflections made per second by the sound wave with the walls of the room

Given : Velocity of sound in air = 350 m/sec

Soln. :

(a) The mean free path of sound waves is defined as the average distance travelled by sound wave through air between any two consecutive encounters with the walls of the room. Jaeger had calculated as

$$l = \frac{4V}{S} = \frac{4\text{ (Volume of the room)}}{\text{Total surface area}}$$

Here $V = 6 \times 4 \times 5 = 120\ m^3$

$$S = 2\,[6 \times 4 + 4 \times 5 + 5 \times 6] = 148\ m^2$$

$$\therefore \quad l = \frac{4 \times 120}{148} = 3.243\ m \qquad \text{...Ans.}$$

Number of reflections made per second

$$n = \frac{\text{Velocity of sound}}{\text{Mean free path}}$$

$$n = \frac{350}{3.243} = 107.9 \qquad \text{...Ans.}$$

Ex. 1.6 : The sound from a drill gives a noise level 90 dB at a point short distance from it. What is the noise level at this point if four such drills are working simultaneously at the same distance from the point ?

Soln. : Acoustic intensity level is given by

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right) \text{ dB} \qquad \text{...(1)}$$

Reference to I_0 in watts / m^2

Let I_1 be the intensity level due to one drill and I_2 be the intensity level due to four such drills.

$$\therefore \quad \frac{I_2}{I_1} = 4 \qquad \text{...(2)}$$

Consider one drill on

$$\therefore \quad L_1 = 10 \log\left(\frac{I_1}{I_0}\right) \text{ dB} \qquad \text{...(3)}$$

In second case with four drills on

$$L_2 = 10 \log\left(\frac{I_2}{I_0}\right) \text{ dB} \qquad \text{...(4)}$$

$\therefore$ Increase in noise level (in dB)

$$L_2 - L_1 = 10\left[\log\left(\frac{I_2}{I_0}\right) - \log\left(\frac{I_1}{I_0}\right)\right]$$

$$= 10 \log\left(\frac{I_2}{I_1}\right)$$

$$\text{but} \quad \frac{I_2}{I_1} = 4$$

$$\therefore \quad L_2 - L_1 = 10 \log 4 = 6.021 \text{ dB}$$

$\therefore$ Final intensity level

$$= L_1 + 6.021 = 90 + 6.021$$

$\therefore$ Final intensity level = 96.021 dB **...Ans.**

Ex. 1.7 : Calculate the increase in the acoustic intensity level in dB. When the sound is doubled.

Soln. :

Intensity level in dB is

$$L = 10 \log \left(\frac{I}{I_0}\right)$$

Let the intensity level in case 1 be I_1 and the in case 2 be I_2

$\therefore$ For case – 1

$$L_1 = 10 \log \left(\frac{I_1}{I_0}\right) \text{dB}$$

$\therefore$ For case – 2

$$L_2 = 10 \log \left(\frac{I_2}{I_0}\right) \text{dB}$$

$\therefore$ Change in intensity level in dB

$$L_2 - L_1 = 10 \left[\log \left(\frac{I_2}{I_0}\right) - \log \left(\frac{I_1}{I_0}\right)\right]$$

$$= 10 \left[\log \left(\frac{I_2}{I_1}\right)\right]$$

but $$\frac{I_2}{I_1} = 2 \text{ (given)}$$

$$\therefore L_2 - L_1 = 10 \log 2$$

$$= 10 (0.3010)$$

$$\therefore L_2 - L_1 = 3.01 \text{ dB}$$ **...Ans.**

Ex. 1.8 : An air conditioner unit operates at a sound intensity level of 70 dB. If it is operated in room with an existing sound intensity level of 80 dB, what will be the resultant intensity level. **(4 Marks)**

Soln. :

Here for case – 1

Intensity level is 70 dB

$$\therefore 70 = 10 \log L_1 = 10 \log \left(\frac{I_1}{I_0}\right)$$

$$\therefore \frac{I_1}{I_0} = \text{Antilog } 7.0$$

or $$I_1 = 10^7 I_0 \text{ watts/m}^2 \qquad \ldots(1)$$

Similarly for Case – 2, intensity level is 80 dB.

$$\therefore 80 = 10 \log L_2 = 10 \log \left(\frac{I_2}{I_0}\right)$$

$$\therefore \frac{I_2}{I_0} = \text{Antilog } 8.0$$

$$\therefore \quad I_2 = 1 \times 10^8 \, I_0 \text{ watts/m}^2 \qquad \ldots(2)$$

$\therefore$ Resultant intensity

$$I = I_1 + I_2$$

$$= 10^7 \, I_0 + 10^8 \, I_0$$

$$= I_0 (1.1 \times 10^8)$$

$\therefore$ Resultant intensity level in dB

$$L = 10 \log \left(\frac{I}{I_0}\right)$$

$$= 10 \log \left(\frac{1.1 \times 10^8 \, I_0}{I_0}\right) = 10 \log (1.1 \times 10^8)$$

$$= 80.41 \text{ dB}$$

$\therefore$ Resultant intensity level (in dB) is 80.41 **…Ans.**

Ex. 1.9 : The noise form an aeroplane engine 100 m from an observer is 40 dB in intensity. What will be the intensity when the aeroplane flies overhead at an altitude of 2 km ?

Soln. : Intensity of sound is given by formula

$$I = \frac{P}{4 \pi R^2}$$

Where P = Acoustic pressure level

R = Radial distance

Here, for case – 1

$$I_1 = \frac{P}{4 \pi R_1^2}$$

And for case – 2

$$I_2 = \frac{P}{4 \pi R_2^2}$$

$$\therefore \quad \frac{I_2}{I_1} = \frac{R_1^2}{R_2^2}$$

Now $R_1 = 100$ m, $R_2 = 2000$ m (given)

$$\therefore \quad \frac{I_2}{I_1} = \frac{100^2}{2000^2} = \frac{1}{400}$$

or

$$\frac{I_1}{I_2} = 400 \qquad \ldots(1)$$

For the case – 1, intensity level in dB is given by

$$L_1 = 10 \log \left(\frac{I_1}{I_0}\right) \qquad \ldots(2)$$

and for case – 2

$$L_2 = 10 \log \left(\frac{I_1}{I_0}\right) \qquad \ldots(3)$$

as intensity level is suppose to decrease, we will take $L_1 - L_2$

$$\therefore L_1 - L_2 = 10\left[\log\left(\frac{I_1}{I_0}\right) - \log\left(\frac{I_2}{I_0}\right)\right]$$

$$= 10 \log\left(\frac{I_1}{I_2}\right)$$

$$= 10 \log 400 = 26.021 \text{ dB}$$

as $L_1 = 40$ dB given

$$\therefore L_2 = L_1 - (L_1 - L_2)$$

$$\therefore L_2 = 40 - 26.021 = 13.97 \text{ dB} \qquad \textbf{...Ans.}$$

Ex. 1.10 : A hall of volume 5500 m^3 is found to have a reverberation time of 2.3 sec. The sound absorbing surface of the hall has an area of 750 m^2. Calculate the average absorption coefficient.

Soln. :

Given : $V = 5500 \text{ m}^3$

$T = 2.3$ sec

$S = 750 \text{ m}^2$

Let absorption coefficient be a

$\therefore$ Using Sabine's formula

$$T = \frac{00.161\ V}{aS}$$

$$a = \frac{0.161\ V}{ST}$$

$$= \frac{0.161 \times 5500}{750 \times 2.3}$$

$$a = 0.513 \qquad \textbf{...Ans.}$$

Ex. 1.11 : For an empty hall of size 20 × 12 × 12 cubic meter, the reverberation time is 2.5 sec. Calculate the average absorption co-efficient of the hall. What area of the floor should be covered by carpet so as to reduce the reverberation time to 2.0 sec. Given that absorption co-efficient of carpet is 0.5.

Soln. :

(a) Reverberation time

$$T_1 = \frac{0.161\ V}{aS} \qquad \text{...(1)}$$

$$\therefore aS = \frac{0.161\ V}{T_1}$$

$$= \frac{0.161 \times (20 \times 12 \times 12)}{2.5}$$

$$= 185.47$$

Now total surface area of the hall,

$$S = 2\,(20 \times 12 + 12 \times 12 + 20 \times 12)$$

$$= 1248 \text{ m}^2$$

$$\therefore a = \frac{185.47}{1248} = 0.1486 \qquad \textbf{...Ans.}$$

(b) By using the carpet of surface area S_1 whose absorption coefficient is 0.5, the reverberation time is reduced to 2.0 sec.

$\therefore$ Let $T_2 = 2.0$ sec

Carpet surface $= S_1$

Co-efficient of absorption of carpet $a_c = 0.5$

$\therefore$ Writing Sabine's formula

$$T_2 = 0.161 \frac{V}{aS + a_C S_1 - aS_1} \qquad \text{...(2)}$$

(Here Total surface area = S, now if carpet is used of area S_1, the area covered by the material with co-efficient of absorption a is a $(S - S_1) = aS - aS_1$)

From Equation (1)

$$\frac{1}{T_1} = \frac{aS}{0.161 \text{ V}} \qquad \text{...(3)}$$

From Equation (2)

$$\frac{1}{T_1} = \frac{aS + a_C S_1 - aS_1}{0.161 \text{ V}} \qquad \text{...(4)}$$

$$\therefore \frac{1}{T_2} - \frac{1}{T_1} = \frac{1}{0.161 \text{ V}} [a_C S_1 - aS_1]$$

$$= \frac{S_1 (a_C - a)}{0.161 \text{ V}}$$

$$\therefore S_1 = \frac{0.161 \text{ V}}{a_C - a} \cdot \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

Substituting various value

$$S_1 = \frac{0.161 (20 \times 12 \times 12)}{0.5 - 0.1486} \left[\frac{1}{2} - \frac{1}{2.5}\right]$$

$$= 131.95 \text{ m}^2$$

$\therefore$ Carpet area required to reduce reverberation time up to 2.0 sec is 131.95 m^2 **...Ans.**

Ex. 1.12 : Calculate the reverberation time for the seminar hall with

(a) No one inside.

(b) 50 persons inside

(c) Full capacity of audience.

Given that

Sr. No.	Surface	Area	Absorption co-efficient

Sr. No.	Surface	Area	Absorption co-efficient
1.	Carpet covering entire floor	(10 × 12) sqm	0.06
2.	False ceiling	(10 × 12) sqm	0.03
3.	Cushioned seats	100 Nos	1.00
4.	Walls covered with absorbent	346 sqm	0.2
5.	Audience occupying seats	–	0.46 / person
6.	Wooden door	(3 × 2) sqm	0.2

Soln. :

Let us calculate total absorption in the hall in case – 1 i.e. for empty hall

(1) Absorption due to carpet $120 \times 0.06 = 7.2$

(2) Absorption due to false ceiling $120 \times 0.03 = 3.6$

(3) Absorption due to seats $100 \times 1 = 100$

(4) Walls covered with absorbent $346 \times 0.2 = 69.2$

(5) Wooden door $6 \times 0.2 = 1.2$

$$\therefore \sum aS = 181.2 \qquad \ldots(1)$$

Now Area of floor $=$ Area of ceiling $= (l \times b)$

$= 120$ sq.m

Area of wall + Area of door $= 346 + 6 = 352$

$= 2\,[(b \times h) + (l \times h)]$

as $l \times b = 120\ m^2$

$\therefore$ let us take $l = 12$ m, $b = 10$ m

$$\therefore 352 = 2\,[(10 \times h) + (12 \times h)]$$

$$\therefore h = 8 \text{ m} \qquad \ldots(2)$$

hence volume $V = 12 \times 10 \times 8 = 960\ m^3$ …(3)

Case 1 :

For empty hall

Reverberation time $T_1 = \dfrac{0.161\ V}{aS}$

$$= \frac{0.161 \times 960}{181.2}$$

$$T_1 = 0.85 \text{ sec} \qquad \textbf{…Ans.}$$

Case 2 :

With occupancy of 50 persons.

$$\therefore \text{Absorption} = aS + 50 \times (0.46)$$

$$\therefore \text{ Reverberation time } T_2 = \frac{0.161\,V}{aS + 50\,(0.46)}$$

$$= \frac{0.161 \times 960}{181.2 + 23}$$

$$T_2 = 0.757 \text{ sec}$$ **...Ans.**

Case 3 :

With full occupancy. i.e. 100 persons here, the absorption is = aS + 100 (0.46)

$$\therefore \text{ Reverberation time } T_3 = \frac{0.161\,V}{aS + 100\,(0.46)}$$

$$T_3 = 0.68 \text{ sec}$$ **...Ans.**

1.11 Solved University Examples :

Ex. 1.11.1 : The volume of room is 600 m^3. The wall area of the room is 220 m^2, the floor area is 120 m^2 and the ceiling area is 120 m^2. The average sound absorption coefficient, (a) for the walls is 0.03 (b) for the ceiling is 0.8 (c) the floor it is 0.06. Calculate the average sound absorption coefficient and the reverberation time.

(Dec. 2008, 3 Marks)

Soln. :

Given :

Let $S_1 = 220 \text{ m}^2 \quad a_1 = 0.03$

$S_2 = 120 \text{ m}^2 \quad a_2 = 0.8$

$S_3 = 120 \text{ m}^2 \quad a_3 = 0.06$

The average sound absorption coefficient is

$$a = \frac{a_1 S_1 + a_2 S_2 + a_3 S_3}{S_1 + S_2 + S_3}$$

$$= \frac{220 \times 0.03 + 120 \times 0.8 + 0.06 \times 120}{220 + 120 + 120} = 0.238$$

$$a = 0.24$$ **...Ans.**

$\therefore$ Total sound absorption of the room = aS

$$= 0.24 \times 460$$

$$= 110.4 \text{ Sabine}$$

Reverberation time, using Sabine's formula

$$T = \frac{0.161\,V}{aS} = \frac{0.161 \times 600}{110.4}$$

$$T = 0.875 \text{ sec.}$$ **...Ans.**

Ex. 1.11.2 : What is the resultant sound level when a 70 dB sound is added to a 80 dB sound ? **(Dec. 2008, 4 Marks)**

(4 Marks)

Soln. : Increase in intensity level = L = 70 dB

Say, resultant intensity increased by x times the original intensity

Hence, $L = 10 \log_{10}\left(\frac{x\, I_o}{I_o}\right)$ dB

$$70 = 10 \log_{10} (x)$$

$$7 = \log_{10} x$$

or $x = 10^7$

So, Resultant sound level is increased 10^7 times the original intensity.

Theory Questions :

Q. 1 Explain the term 'acoustics'.

Q. 2 Explain 'Reverberation' and 'Reverberation time'.

Q. 3 Write the difference between sound intensity and loudness

Q. 4 What are the units used to measure sound intensity ? Explain them.

Q. 5 Define the term 'coefficient of absorption' and write a short note on it.

Q. 6 What is unit associated with absorption coefficient ?

Q. 7 Explain the terms :

(a) Reflection

(b) Reverberation

(c) Echo of sound energy and then show graphically the nature of growth and decay of sound energy in a hall due to reverberation.

Q. 8 Explain how the reverberation time of a hall is affected by

(a) Size

(b) Nature of its wall surfaces and

(c) Audience

Q. 9 Write down Sabine's formula for vibration time. Explain terms involved in it with their units. What are the limitations of Sabine's formula ?

Q. 10 What do you mean by 'acoustically correct hall' ? Or what are the conditions for 'good acoustics' ?

Q. 11 Explain various remedies suggested for good acoustics.

Q. 12 What is echelon effect ?

Assignments :

Q. 1 A class room has dimensions $20 \times 15 \times 5$ m^3. The reverberation time is 3.5 sec. Calculate the total absorption of its surface and average absorption co-efficient.

Ans. : (0.07)

Q. 2 The reverberation time is found to be 1.5 sec for an empty hall and it is found to be 1 sec when a curtain of 20 m^2 is suspended at the center of the hall. If the dimensions of the hall are $10 \times 8 \times 6$ m^3, calculate co-efficient of absorption of curtain.

Ans. : (0.64)

Q. 3 For an empty assembly hall of size $20 \times 15 \times 10\ m^3$, the reverberation time is 3.5 sec. Calculate the average absorption co-efficient of the hall. What area of the wall should be covered by the curtain so as to reduce the reverberation time to 2.5 sec. Given the absorption co-efficient of the curtain cloth is 0.5.

Ans. : (0.106, 140.12 m^2)

Multiple choice Questions :

Q. 1 Velocity of sound depends upon following characteristic of medium :

(a) Length (b) Temperature

(c) Mass (d) Non of the above **(Ans. : (b))**

Q. 2 Frequency of the following is less than audible waves :

(a) Infrasonic (b) Ultrasonic

(c) Supersonic (d) Non of the above **(Ans. : (a))**

Q. 3 If 'S' is loudness and intensity is denoted by 'I', then $\frac{ds}{dI}$ represents.

(a) Arbitrariness (b) Sensitiveness

(c) Roughness (d) Bel **(Ans. : (b))**

Q. 4 Threshold of hearing has its intensity level.

(a) 10 dB (b) 50 dB

(c) 100 dB (d) 0 dB **(Ans. : (d))**

Q. 5 Sound level with its intensity level greater then 120 dB is due to.

(a) Normal conversation (b) Whisper

(c) Guitar (d) Painful sound **(Ans. : (d))**

Q. 6 If the intensity level of sound changes by 100 times its original the increase in dB is :

(a) 10 dB (b) 100 dB

(c) 20 dB (d) 40 dB **(Ans. : (c))**

Q. 7 The ideal absorber of the sound :

(a) Open window (b) Carpet

(c) Heavy curtain (d) Perforated cellulose fiber files **(Ans. : (a))**

Q. 8 Reverberation time is :

(a) Directly proportional to volume

(b) Inversely proportional to volume

(c) Equal to volume

(d) None of the above **(Ans. : (a))**

Q. 9 The preferred one for acoustically correct auditorium

(a) Parallel walls

(b) Splayed walls

(c) Curved walls

(d) White wall **(Ans. : (b))**

Q. 10 Following is merely a sensation :

(a) Loudness (b) Intensity

(c) Reverberation time (d) Bel **(Ans. : (a))**

Fill-up the blank position :

1. Sound waves are ----------- in nature. **(Ans. : Longitudinal)**
2. Sound waves can not pass through --------------. **(Ans. : Vacuum)**
3. Audible sound wave have their frequency range -----------. **(Ans. : 20 Hz to 20 KHz)**
4. According to Weber-Fechner law the loudness is directly proportional to ----------.

   (Ans. : Natural logarithm of it's intensity)
5. When intensity of sound increases by a factor of 10, the increase in the intensity is said to be -------------. **(Ans. : 1 bel)**
6. When intensity of sound changes by 26%, the intensity level alters by -------------.

   (Ans. : 1 dB)
7. The time gap between the initial direct note and the reflected note upto a minimum audibility level is ---------------. **(Ans. : reverberation time)**
8. The absorption coefficient is measured in -------------. **(Ans. : open window unit)**
9. Echelon effect is produced by -----------. **(Ans. : Staircase)**
10. Use of absorbents ------------- the reverberation time. **(Ans. : decreases)**

1.12 University Questions and Answers :

Dec. 2008

Q. 1 What is standard intensity ? Give its value. **(Section 1.2.1)** **(1 Mark)**

Q. 2 Define reverberation time. **(Section 1.4)** **(1 Mark)**

Q. 3 The volume of the room is 600 m^3 the wall area of the room is 220 m^2 the floor area is 120 m^2 and ceiling area is 120 m^2 . The average sound absorption coefficient for wall is 0.03, for ceiling is 0.8 and for floor it is 0.06. Calculate reverberation time.
(Ex. 1.11.1) **(3 Marks)**

Q. 4 What is meant by time of reverberation ? Discuss Sabine's Formula.
(Section 1.6) **(5 Marks)**

Q. 5 State any five factors affecting the acoustics of the building and give at least two remedies for each. **(Section 1.9)** **(5 Marks)**

Q. 6 What is the resultant sound level when a 70 dB sound is added to a 80 dB sound ?

(Ex. 1.11.2) **(4 Marks)**

June 2009

Q. 7 List and explain the characteristics of musical sound. **(Section 1.1.1)** **(3 Marks)**

Q. 8 Explain factor affecting acoustics of the building. **(Section 1.9)** **(3 Marks)**

NOISE POLLUTION

Noise is defined as unwanted sound. It is considered an environmental pollutant because, if it is present at sufficient intensity, undesired physiological and psychological effects are produced. Unlike chemical pollution, noise pollution can be immediately removed from the environment. Moreover, noise pulses have much shorter decay times than hazardous chemical shocks. However, once the disturbance begins, a reduction in the severity of its effect may be very difficult in contrast to a chemical plant upset. Simple action by stopping a leak, diverting a hazardous liquid flow, or shutting down equipment may prevent further environmental damage in that case, while noise amelioration may require redesign or a whole enclosure.

High intensity noise causes temporary or permanent hearing loss, interference with speech communication, sleep, and relaxation, and an impaired ability to carry out basic tasks. Prolonged exposure to noise may result in a slow loss of hearing over many years. The affected individual may be unaware that he or she is being injured by the noise until the hearing impairment becomes known. Because of the long apparent latency period, it becomes difficult to determine how long injury has taken place and to define the time-intensity noise profile causing the injury.

With the growth of modern society, noise levels have steadily risen, creating various adverse effects on our quality of life. Laws regarding noise control have been developed, requiring that noise be controlled in cities near airports, railway stations, freeways, and in other public areas. Air traffic routes and freeway sound barriers are designed to minimize disturbances to homeowners. Industrial operations must also comply with noise regulation. Areas within factories or chemical plants where noise levels are high have warning signs indicating the need for noise protection (ear plugs). Noise surveys are routinely done to protect workers and to satisfy occupational health and safety requirements.

This chapter presents the basic fundamentals of noise pollution and control. There are a growing number of environmental engineering books covering noise pollution. It is likely that, as new noise control technologies are developed and the regulations become more stringent, this area will gain additional importance to the environmental engineering profession.

PROPERTIES OF NOISE

Sound waves create sinusoidal changes in air pressure. The surrounding air undergoes an alternating compression and expansion causing a cyclic increase and decrease in air pressure and density. The time between the highs and lows in air pressure is known as the *period*. The *frequency* is the inverse of the period:

$$f = 1/P$$

and is measured in the units of Hertz (Hz). Sound waves are characterised by *wavelength*, or distance between the peaks or troughs and *amplitude*, the height of the peaks and troughs relative to a zero pressure line. Sound measurements are expressed in terms of root mean square air pressure:

$$P_{rms} = \sqrt{\overline{P^2}} = \sqrt{\frac{1}{T}\int_0^T P^2(t)dt}$$

where P is the air pressure and T is the time. The bar indicates that the pressure is a time weighted mean quantity. Use of an average pressure for sound measurement would not be possible because it averages to zero. Squaring also leads to adding, not subtracting negative values.

The time weighted average pressure force exerted by sound over the distance of propagation of the sound waves is *work*. The rate at which the work is done is the *sound power* (W). *Sound intensity* is the sound power per unit area normal to the direction of propagation.

Sound power is related to sound pressure:

$$I = (P_{rms})^2/\rho c$$

where I = sound intensity, W/m^2
P_{rms} = Root means square pressure
ρ = density of air
c = speed of sound

At 1 atmosphere:

$$c = 20.05\sqrt{T}$$

where T is in degrees K and c is in m/s.

SOUND LEVELS

The faintest sound detectable by the human ear in terms of air pressure is 0.00002 Pa (20 μPa) and that caused by a Saturn rocket is 200 Pa. Because of this enormous range, a scale based on logarithm of ratios of measured quantities is used. The units are referred to as Bels;

$$L' = \log Q/Q_o$$

where L' = level, Bels
Q = measured quantity
Q_o = reference quantity

Log = logarithm in base 10

The Bel is normally subdivided into decibels (dB):

$$L = 10 \log Q/Q_O$$

For sound the measured quantity, Q, may be *sound power*, *sound intensity*, and *sound pressure*. The reference levels, Q_O, for sound power and sound intensity are 10^{-12} W and 10^{-12} W/m^2, respectively. The sound pressure level is computed as:

$$L_p = 10 \log(P_{rms})^2/(P_{rms})_O{}^2$$

or equivalently;

$$L_p = 20 \log P_{rms}/(P_{rms})_O$$

The reference level is 20 μPa. The relative scale of sound pressure levels is shown in Figure 1.

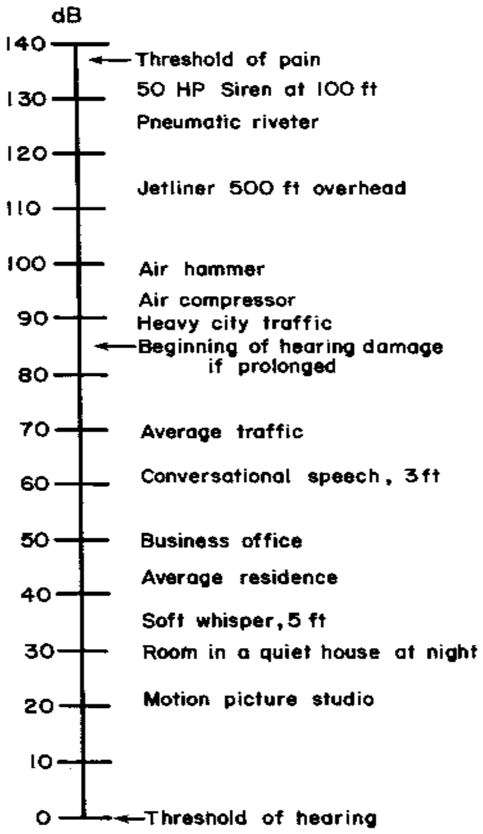


Figure 1 Environmental Noise

From Vesilind, P.A., Peirce, J.J., and Weiner, R.F. 1990. *Environmental Pollution and Control.* Butterworth-Heinemann, Figure 22-4 p 333

The addition of sound in decibels is illustrated in the following example.

Example 1

Determine the noise level corresponding to the addition of two noise levels of 60 dB.

The ratio of root mean square pressures for 60 dB is:

$$(P_{rms})^2/(P_{rms})_O{}^2 = 10^{60/10}$$

The sum of root mean square pressures for two sources is

$$P^2_{rms1+2} = P^2_{rms1} + P^2_{rms2}$$

Therefore:

$$60\text{ dB} + 60\text{ dB} = 10\log(10^{60/10} + 10^{60/10}) = 63\text{ dB}$$

The sound pressure level is reported as 63 dB re: 20 μPa

Note that the doubling of sound level causes a 3dB gain. This is often the limit of what is admissible for new activities.

Adding more sound sources, e.g. 68, 75 and 79 dB, would just be 10 log ($10^{68/10} + 10^{75/10} + 10^{79/10}$) = 80.7dB

The human sensitivity to sound depends not only on sound pressure but also on sound frequency. The frequency range of sound can be estimated using a sound level meter, which contains electronic filtering circuits, which approximate the human response to sound at various frequencies. The weighting characteristics of a sound level meter are A, B, and C. Low sound frequencies are filtered substantially by the A network, moderately by the B network, and hardly at all by the C network. The response characteristics of the three weighting networks are shown in Figure 2. The filters subtract or add dB at the various frequencies as illustrated in Table 1. The readings (dB) for each of the three networks over the sound level frequency range are then added by logarithmic addition (as in the previous example) and reported as sound levels.

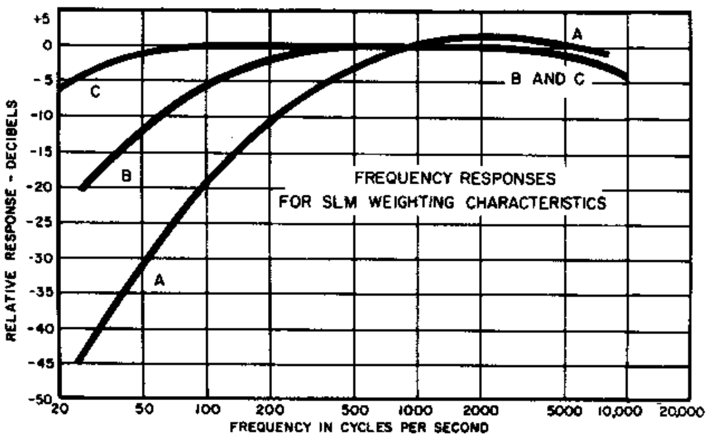


Figure 2 The A, B, and C Filtering Curves for a Sound Level Meter
From Vesilind, P.A., Peirce, J.J., and Weiner, R.F. 1990. *Environmental Pollution and Control.* Butterworth-Heinemann.Figure 23-3 p 341

Example 2

A Type 2 sound level meter is to be tested with two pure tone sources that emit 90 dB. The tones are 1000 and 125 Hz. Estimate the expected readings on the A, B and C weighting system.

In the low frequencies (200 to 1000 Hz) the filters subtract much more from the A network than the B and C networks. At 1000 Hz, there is a zero correction, so the reading for all three should be 90 dB. At 125 Hz, the values will be 90 –16.1 = 74 dBA, 86 dBB and 90 dBC.

Table 1   Network Weighting Values for Sound Level Meter

From: Davis, M.L., and D.A. Cornwell. 1991. *Introduction to Environmental Engineering*. McGraw-Hill, Inc. Table 7-1 p 511.

Frequency (Hz)	Curve A (dB)	Curve B (dB)	Curve C (dB)
10	-70.4	-38.2	-14.3
12.5	-63.4	-33.2	-11.2
16	-56.7	-28.5	-8.5
20	-50.5	-24.2	-6.2
25	-44.7	-20.4	-4.4
31.5	-39.4	-17.1	-3.0
40	-34.6	-14.2	-2.9
50	-30.2	-11.6	-1.3
63	-26.2	-9.3	-0.8
80	-22.5	-7.4	-0.5
100	-19.1	-5.6	-0.3
125	-16.1	-4.2	-0.2
160	-13.4	-3.0	-0.1
200	-10.9	-2.0	0
250	-8.6	-1.3	0
315	-6.6	-0.8	0
400	-4.8	-0.5	0
500	-3.2	-0.3	0
630	-1.9	-0.1	0
800	-0.8	0	0
1,000	0	0	0
1,250	0.6	0	0
1,600	1.0	0	-0.1
2,000	1.2	-0.1	-0.2
2,500	1.3	-0.2	-0.3
3,150	1.2	-0.4	-0.5
4,000	1.0	-0.7	-0.8
5,000	0.5	-1.2	-1.3
6,300	-0.1	-1.9	-2.0
8,000	-1.1	-2.9	-3.0
10,000	-2.5	-4.3	-4.4
12,500	-4.3	-6.1	-6.2
16,000	-6.6	-8.4	-8.5
20,000	-9.3	-11.1	-11.2

Octave band analysis is frequently done for community noise control. The noise is broken down into 8 to 11 frequency intervals (octaves) with the highest frequency of the interval being twice the lowest frequency. (An octave jump in music also signifies a doubling or halving in frequency.) The analysis is done with a sound level meter (combination precision) and an octave filtering set.

Another unit of importance is the *phon,* which is a measure of *loudness*, the brain's perception of the magnitude of sound levels. The phon is the loudness of a tone that is numerically equal to the corresponding sound pressure level when heard at 1000 Hz. A sound pressure level of 40 dB when heard at 1000-Hz is 40 phons.

Averaging sound pressure levels

Average sound pressure level, $L_p = 20 \log 1/N \ \Sigma\ 10^{Lj/20}$

Where L_j is the jth sound pressure level, dB re: 20μPa

Example 3: Compute the mean sound level from the following sounds: 40, 52, 68 and 78 dBA.

$\Sigma = 10^{40/20} + 10^{52/20} + 10^{68/20} + 10^{78/20} = 1.1 \times 10^4$

$L_p = 20 \log (1/4 \times 1.1 \times 10^4) = 69$dBA

EFFECTS OF NOISE

The human ear is extremely sensitive to sound. The lowest perceptible sound pressure in the frequency range of speech (500 to 2000 Hz) is 20 μPa. This may be compared with the 1 μPa sound pressure corresponding to the thermal motion of air molecules. Our perception of sound is controlled by the human auditory system consisting of the outer, middle, and inner ear. Noise-related injury may result in damage to the eardrum but more often damage occurs to the tiny hair cells in the inner ear. The former is the result of very loud, sudden noises and the latter the result of chronic exposure to noise. A loss of hearing is quantified as a *threshold shift,* which may be either temporary or permanent.

Rock band members are sometimes victims of a *temporary threshold shift* (TTS). In one study members of a band suffered a 15 dB TTS after a concert. *Permanent threshold shift* (PTS) has been reported for workers in a textile mill where noise levels were highest (4000 Hz). Figures 3 and 4 illustrates that the severity of a temporary or permanent injury depends on the frequency of the noise at which it is assessed.

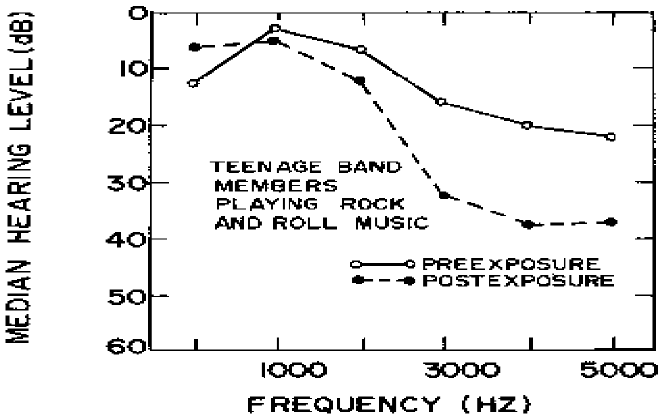


Figure 3 Temporary Threshold Shift for Rock Band Performers

From Vesilind, P.A., Peirce, J.J., and Weiner, R.F. 1990. *Environmental Pollution and Control.* Butterworth-Heinemann.Figure 22-7 p 336

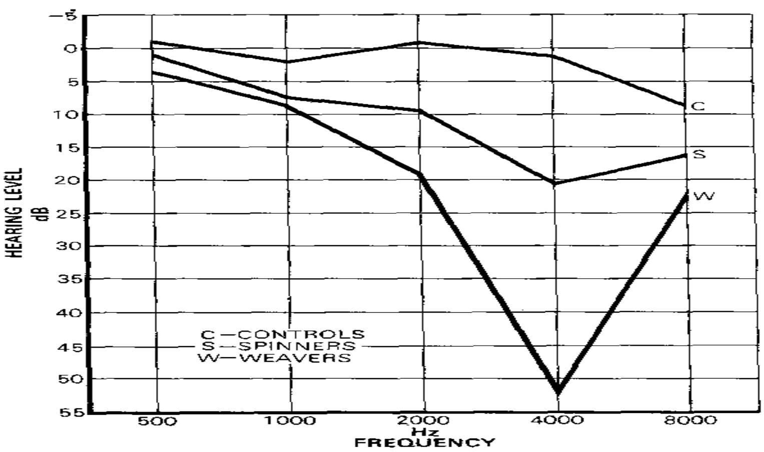


Figure 4 Permanent Threshold Shift for Textile Workers.
From Vesilind, P.A., Peirce, J.J., and Weiner, R.F. 1990. *Environmental Pollution and Control.* Butterworth-Heinemann.Figure 22-8 p 337

NOISE RATING SYSTEM

A rating system is developed to assess the daily impact of noise on the surroundings. The impact of noise is related not only to sound pressure but also to frequency, and whether the noise is continuous, intermittent, or impulsive. Two systems commonly used in rating noise are referred to as the L_N and the L_{eq} concepts. The L_N concept is a statistical measure of how frequently a particular sound level is exceeded. For instance, L_{40} = 72 dB(A) implies that 72 dB(A) was exceeded 40% of the measuring time. A plot of L_N against N has the appearance of a cumulative distribution curve (Figure 5).

The equivalent continuous equal energy level, Leq, is calculated as

$$L_{eq} \quad = \quad 10 \log 1/t \int_0^t 10^{L(t)/10}\, dt$$

where t = the time over which L_{eq} is determined and L(t) = the time varying noise level in dBA.

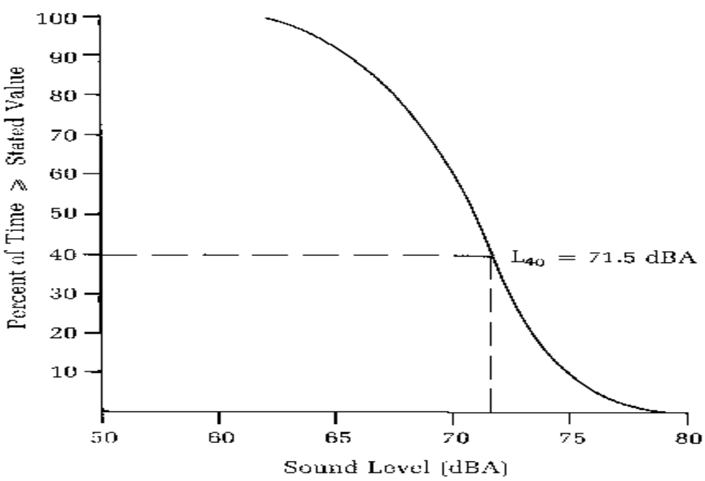


Figure 5 Cumulative Distribution Curve
From: Davis, M.L., and D.A. Cornwell. 1991. *Introduction to Environmental Engineering*. McGraw-Hill, Inc. Figure 7-23 p 532.

The working version of the equation, for discrete samples, is

$$L_{eq} \quad = 10 \log \sum_{i=1}^{i=n} 10^{Li/10} t_i$$

where n = total number of samples taken
L_i = noise level in dBA of the ith sample
t_i = fraction of total sample time

Example 3

Calculate the L_{eq} corresponding to a 90 dBA noise for 5 minutes followed by a 60 dBA noise for 50 minutes. Assume a sampling interval of 5 minutes.

The t_is for the 5 minute and 50 minute samples are 1/11 (0.091) and 10/11 (0.91). The sum is:

$(10^{90/10})(0.091) + (10^{60/10})(0.91) = 9.19 \times 10^7$

Therefore: Leq = $10 \log(9.19 \times 10^7)$ = 80 dBA

NOISE SOURCES AND CONTROL

The following are common noise sources, which may represent an annoyance to members of a community.

- aircraft
- rail traffic
- highway vehicles
- internal combustion engines
- industrial activities
- stadiums
- construction activities

The U.S. Federal Highway Administration (FHA) has developed noise standards for various land use categories. For areas where serenity and quiet are necessary such as amphitheatres, parks, or open spaces the L_{eq} and L_{10} values averaged over 1 hour are 57 and 60, respectively. Residences, motels, schools, churches, and recreational areas should have L_{eq} and L_{10} not exceeding 67 and 70, respectively.

The quantification of the noise level in a community causing annoyance is difficult. Aircraft noise associated with overflying aircraft can represent a significant disturbance to a residential neighbourhood, particularly if landing and takeoff patterns are over residential areas. Internal combustion (IC) engines used around the home such as lawn mowers, chain saws, and model aircraft cause sporadic annoyance but are, in general, not considered to be significant contributors to community noise. The dBA range at 15.2 m for residential IC engines is 70-84. Noise associated with single house construction in a suburban community will generate sporadic complaints if the 8 hour L_{eq} exceeds 70 dBA. Threats of legal action are common for major construction activities in suburban communities with an 8-hour L_{eq} greater than 85 dBA.

Control of noise requires some understanding of three basic elements 1) the nature of the source, 2) the path over which the noise travels, and 3) the receiver or listener. Solution to the problem may require examining how each might be altered to reduce noise. Three possibilities for alleviation of the problem are

1. Reduction of the noise output of the source
2. Change the transmission path of the noise to reduce the noise level
3. Provide the listener with protective equipment.

Noise at the *source* (equipment) is decreased by reduction of impact forces, frictional resistance, speeds and pressures, noise radiating area, and noise leakage. Mufflers or silencers also reduce noise.

If noise reduction is not possible at the source, the use of devices to block or reduce the sound along its *transmission path* may reduce the effect of the noise. Three ways of accomplishing this are 1) absorption of the sound along its path, 2) deflection of the sound, and 3) contain the source in a sound-insulating enclosure. The atmosphere has absorption capacity for sound. Doubling the distance from a point source reduces its sound pressure level by 6 dB. For a line source such as a train the sound reduction with doubling of distance is 3 dB. Sound absorbing materials including drapes, acoustical tiles and carpets have the potential to reduce the sound pressure level in a room by 5-10 dB for high frequency sounds and 2-3 dB for low frequency sounds. Lining the inside surfaces of ducts, pipe chases, or electrical channels with sound-absorbing materials reduces noise. A reduction of 10 dB/m of high-frequency noise in a duct installation with an acoustical lining of 2.5 cm thick is not uncommon. However, reduction of low frequency noise requires thicker and longer acoustical lining.

Barriers, screens, and deflectors are effective in reducing noise transmission. The size of the barrier and frequency of noise are important parameters influencing the efficiency of noise reduction. High frequency noise is attenuated to a greater extent than low frequency noise. Noise passing well over the top of a barrier to receptors, who can see the source is unattenuated. However, noise just passing over the top of barrier is diffracted downward to the other side at an angle inversely proportional to the sound energy passing over the barrier. Noise will also be transmitted directly through the barrier. Finally, the noise may be reflected to the opposite side of the source. The diffracted noise is considered to be the most important for design of barriers.

When considering the design of barriers, it is useful to relate the reduction in dB to energy and loudness reduction. This is illustrated for a line source in Table 2.

Table 2 Noise Reduction with Barriers

Reduction in A-level dB	% Reduction in energy	% Loudness reduction
3	50	17
6	75	33
10	90	50
30	99.9	88
40	99.99	93.8

It is apparent that a reduction in noise of 10 dB(A) provides substantial reduction in loudness but requires that the sound energy be reduced by 90% which would require a very long and high barrier. The complexity of barrier design varies from simple for a 10 dB reduction to nearly impossible for a 20 dB reduction. The noise reduction for various highway configurations is shown in Table 3. Increasing the number of trucks on the highway decreases the noise reduction. Noise reduction is less at 152 m because of limited "shadow" of the barrier.

When the noise level cannot be reduced, protection to the receiver is possible by providing

1. limited exposure to the disturbing noise source
2. curtailed noisy operations at night and early morning to avoid disturbance of sleep
3. ear protection

Ear protection devices such as moulded and pliable earplugs, cup-type protectors, and helmets provide noise reduction ranging from 15 to 35 dB. However, they do have disadvantages which include the interference with speech communication and the hearing of warning calls. Therefore, it is recommended that they be used if the other control measures such as source and transmission reduction are not possible.

Highway configuration[a]		Height or depth (m)	Truck mix (%)	Noise reduction[b] at distance from ROW (dBA)	
Sketch	Description			30 m	152 m
	Roadside barriers 7.6 m from edge of shoulders; ROW = 78 m wide	6.1	0	13.9	13.3
			5	13.0	12.1
			10	12.6	11.7
			20	12.3	11.3
	Depressed roadway w/2:1 slopes; ROW = 102 m	6.1	0	9.9	11.4
			5	8.8	10.3
			10	8.4	9.8
			20	8.1	9.4
	Fill elevated roadway w/2:1 slopes; ROW = 102 m	6.1	0	9.0	6.3
			5	7.6	2.7
			10	7.1	1.8
			20	6.7	1.1
	Elevated structure; ROW = 78 m	7.3	0	9.8	6.0
			5	9.6	2.4
			10	9.3	1.5
			20	8.8	0.8

[a] Assumes divided 8 lanes with 9.1 m median.

[b] Based on observed 1.5 m above grade.

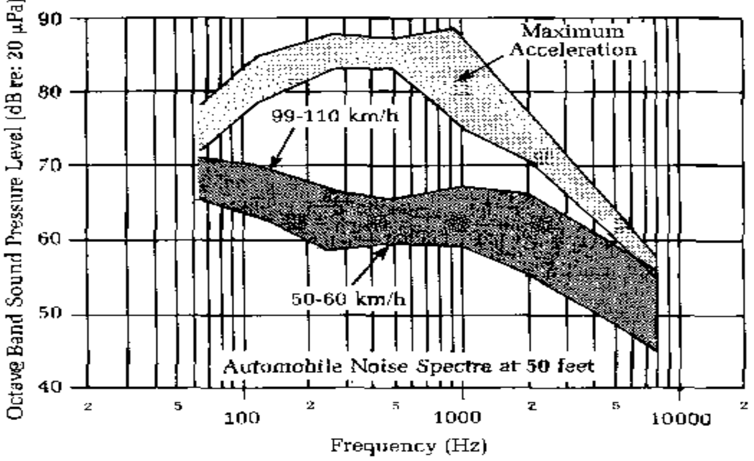
Source. B. A. Kugler, D. E. Commins, W. J. Galloway, "Highway Noise: Generation and Control," *National Cooperative Highway Research Program Report 173*, 1976.

Table 3 Noise Reductions for Various Highway Configurations

From: Davis, M.L., and D.A. Cornwell. 1991. *Introduction to Environmental Engineering*. McGraw-Hill, Inc.Table 7-10

PROBLEMS

1. The figure below represents a typical noise spectrum for automobiles travelling at 50 to 60 km/h. Determine the equivalent A-weighted level using the following geometric mean frequencies for octave bands of 63, 125, 250, 500, 1,000, 2,000, 4,000, and 8,000. Estimate the dB readings corresponding to each of the frequencies.



From: Davis, M.L., and D.A. Cornwell. 1991. *Introduction to Environmental Engineering*. McGraw-Hill, Inc. Figure 7-27 p 536.

2. A motorcyclist is warming up her racing cycle at a racetrack approximately 200m from a sound level meter. The meter reading is 56 dBA. What meter reading would you expect if 15 other motorcyclists join her with motorcycles having exactly the same sound emission characteristics? Assume that all of the motorcycles are located at the same point.

3. You are working in a production plant and are required to conduct a noise survey. You will also make recommendations regarding how plant operations/equipment might be modified to improve the current situation. Discuss how you would conduct the survey and, based on the results, what improvements you would recommend. Consider hypothetical noise conditions as examples on which to base your recommendations.

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